Bayesian 3-D interferometric ISAR imaging for the targets with limited pulses

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Abstract: It is difficult to yield high azimuth resolution of a target in short coherent processing interval, which brings certain challenges for the subsequent three-dimensional (3-D) interference inverse synthetic aperture radar (InISAR) imaging. Aiming at this problem, this study introduces a novel 3-D InISAR imaging algorithm for the target with limited pulses. It is assumed that the received echoes have been compensated and image co-registered, and the variational Bayesian interference can be used to process the received limited pulses to yield the high-resolution 2-D ISAR images. At last, the spatial 3-D image of the target can be reconstructed after combining the interferometric processing and range measurement. The simulation results substantiate the validity of the presented algorithm.

1 Introduction

Three-dimensional (3-D) interference inverse synthetic aperture radar (InISAR) imaging has been widely studied because it can reconstruct the geometric distribution of the target via the phase difference in multiple 2-D ISAR images to extract the coordinate information of the scatterers [1–3]. In order to reconstruct the target’s 3-D image as completely as possible, it is required that the scatterers on the target should be resolved as much as possible within the imaging projection plane, which means the high-resolution ISAR images are needed. As we all know, the high resolution of the range dimension can be achieved by using a large bandwidth signal; the azimuth dimension is obtained by accumulating pulses in a long coherent processing interval [4–6]. However, there will be some problems: (i) The echoes of manoeuvring target received during the long coherent processing interval have obvious time-varying Doppler modulation which is difficult to yield high-quality 2-D ISAR imaging. (ii) The large amount data needs to be processed during the long coherent processing interval, which reduces the calculation efficiency. Therefore, how to obtain the 2-D high-resolution images of the target with limited pulses becomes a major problem in 3-D InISAR imaging.

Nowadays, with the development of compressive sensing (CS) technology, the unknown sparse signals can be reconstructed with high precision even when the received echoes are limited. Since the azimuth echoes of the non-cooperative target is always sparse in Doppler domain, it is possible to yield high-resolution ISAR images of the target with limited pulses by using the CS algorithms. [7–10] proposed a series of CS algorithms (such as OMP, SL0, Bayesian etc.) to yield high-quality 2-D images of a target with limited or missing echoes. Based on this, [11] proposed a modified orthogonal matching pursuit decomposition approach to solve the joint-sparsity constraint optimisation problem to realise 2-D ISAR imaging of the sparse measurements; the 3-D InISAR image is reconstructed by using the ISAR images. [12, 13] both model the reconstruction of the super-resolution multi-channel ISAR images as a joint sparse constrained optimisation problem, and the conjugate gradient method and Bayesian CS technique have been used to solve the problem, respectively; then, the 3-D reconstructed images of a target with limited echoes are obtained after interferometric processing. Different from [12, 13], the echoes of each radar are modelled as an optimisation problem after using the algorithms in [14, 15] to perform motion compensation and image co-registration in this paper; then, we use the variational Bayesian inference to solve these problems to achieve high-resolution 2-D ISAR imaging. Compared with the SBL algorithm in [16], the variational Bayesian will obtain better imaging results because it increases the constraints on noise variance in the model. By the interference processing of the ISAR images, the interference phases are obtained, and the 3-D InISAR image of a target with limited pulses could be achieved after the coordinate transformation by using the interference phases ultimately. The simulation results validate the applicability for the algorithm.

2 Introduction of the 3-D InISAR imaging theory

As shown in Fig. 1, we assume that the space radar 3-D coordinate system \((O, U, V, W)\) is constructed with \(O\) as the origin, and the ship target moves in the plane \(UOV\). The radars \(A, B, C\) are located at the origin \(O\), \(U\), and \(W\) axes, respectively, where \(A\) is the transmitter and both \(B\) and \(C\) are the receivers. The lengths of the baselines \(AB\) and \(AC\) are all \(L\), and the distance from an arbitrary scatterer \(p\) on the ship target to radar \(A, B\) and \(C\) are \(R_{pA}, R_{pB}\) and \(R_{pC}\), respectively. It is supposed that the transmitter transmits a linear frequency modulated (LFM) signal with a high time-bandwidth product as

\[
S(t, t_m) = \Omega \exp\left(\frac{j2\pi f_t + \frac{3}{2}\gamma t^2}{2}\right) \quad |\tilde{t}| < \frac{\tau}{2}
\]

where \(\tilde{t} = t - t_m = t - mT_c\) is the fast time, \(m = 1, 2, \ldots, M\), where \(M\) is the total pulse accumulation number. \(t\) is the electromagnetic wave propagates, \(t_m\) is the slow time, and \(T_c\) is the pulse repetition period. The magnitude of the LFM signal is \(\Omega\), \(f_t\) denotes the carrier frequency, and \(\gamma\) is the frequency modulation rate. Supposed that the three radar echoes are dechirped by using the demodulation frequency technology, then the expressions of the dechirped echoes are written as
where $\delta_0$ is the back scattering magnitude of the scatterer $p$, $p = 1, 2, \ldots, P$. $\Gamma$ is the whole number of the scatterers. $R_{\rho}(t_m)$, $\Gamma = A, B, C$ is the distance between $p$ and radar $\Gamma$, and we choose the distance measurement of the target at radar $\Lambda$ as the reference distance $R_{\text{ref}}$ customarily. The last term in (2) which has nothing to do with the ISAR imaging can be ignored for the far-field conditions, or some compensation measures can be taken to eliminate this item. Then, the 1-D range envelopes of the target at radar $\Gamma$ can be yielded by performing range compression on (2), and the expressions of the range profiles are described as

$$
S_\rho(r, t_m) = \sum_p \delta_0 \Omega \sin \left(-\frac{2\pi}{c} f_r (R_{\Lambda \rho}(t_m) + R_{\rho \rho}(t_m) - 2R_{\text{ref}})\right) \times \exp \left[-j\frac{2\pi}{c} f_r (R_{\Lambda \rho}(t_m) + R_{\rho \rho}(t_m) - 2R_{\text{ref}})\right] \times \exp \left[-j\frac{2\pi}{c} f_r (R_{\Lambda \rho}(t_m) + R_{\rho \rho}(t_m) - 2R_{\text{ref}})\right]$$

(2)

To (3), it is easy to find that the 1-D range envelopes are defocused in the range domain because of the distance $R_{\Lambda \rho}(t_m) + R_{\rho \rho}(t_m)$ is changing with the slow time when the target moves, which will bring some challenges to the ISAR imaging. Fortunately, there are lots of mature compensation algorithms are proposed to eliminate the bad influence brought by the changes recently, and we choose the algorithms in [14] to compensate (2) in this paper. According to [11], the new 1-D range envelopes of radars $A, B$, and $C$ are updated to

$$
S_\rho(r, t_m) = \sum_p \delta_0 \Omega \sin \left(-\frac{2\pi}{c} f_r (r - \frac{R_{\rho \rho}(t_m) + R_{\rho \rho}(t_m)}{2})\right) \times \exp \left[-j\frac{2\pi}{c} f_r (R_{\rho \rho}(t_m) + R_{\rho \rho}(t_m) - 2R_{\text{ref}})\right]
$$

(3)

From (3), it is easy to find that the 1-D range envelopes are defocused in the range domain because of the distance $R_{\Lambda \rho}(t_m) + R_{\rho \rho}(t_m)$ is changing with the slow time when the target moves, which will bring some challenges to the ISAR imaging. Fortunately, there are lots of mature compensation algorithms are proposed to eliminate the bad influence brought by the changes recently, and we choose the algorithms in [14] to compensate (2) in this paper. According to [11], the new 1-D range envelopes of radars $A, B$, and $C$ are updated to

$$
S_\rho(r, t_m) = \sum_p \delta_0 \Omega \sin \left(-\frac{2\pi}{c} f_r (r - r_p)\right) \exp \left[-j\frac{4\pi}{c} f_r v_p t_m\right] \Phi_\rho
$$

(4)

where $r_p$ is the radial distance of the scatterer $p$, $v_p$ is the equivalent radial velocity after motion compensation, $\Phi_\rho(t_m)$ is the phase for the radar $\rho$, which is illustrated as

$$
\Phi_\rho(t_m) = \exp \left(-j4\pi f_r v_p t_m/c\right)
$$

(5)

$$
\Phi_\rho(t_m) = \Phi_\rho(t_m) \exp \left(j2\pi f_r \Gamma_\rho(t_m)/c\right) \Gamma = B, C
$$

(6)

where the second term in (6) is the phase differences caused by the location diversity of the three radars, which is necessary for 3-D interference ISAR imaging. From [11], we can expand $\Gamma_\rho(t_m)$ into Taylor series as

$$
\Gamma_\rho(t_m) \approx \Gamma_{\rho 0} + \Gamma_{\rho t} t_m + \Gamma_{\rho r} r_p + \cdots
$$

(7)

At this point, after taking (5) (6) (7) into (4), the form of the 1-D range profiles can be obtained as follows

$$
S_\rho(r, t_m) = \sum_p \Theta_\rho \exp \left[-j\frac{4\pi}{c} f_r v_p t_m\right] \exp \left[-j\frac{4\pi}{c} f_r v_p t_m\right] \times \exp \left[-j\frac{4\pi}{c} f_r (r_p - \frac{1}{2} \Gamma_{\rho 0})\right] \exp \left[j\frac{4\pi}{c} f_r \Gamma_\rho(t_m)\right]
$$

(8)

$$
S_\Gamma(r, t_m) = \sum_p \Theta_\rho \exp \left[-j\frac{4\pi}{c} f_r v_p t_m\right] \times \exp \left[-j\frac{4\pi}{c} f_r (r_p - \frac{1}{2} \Gamma_{\rho 0})\right] \exp \left[j\frac{4\pi}{c} f_r \Gamma_\rho(t_m)\right]
$$

(9)

where $\Theta_\rho = \delta_0 \Omega \sin (c - 2\gamma/c(r_p - v_p))$, the second and third terms in (8) and (9) are used for azimuth compression and interference

ISAR imaging, respectively. The last term in (9) should be eliminated because it may cause the received three ISAR images mismatch when the signals are compressed in the azimuth time field, resulting in the misalignment for the interference scatterers. A joint phase autofocus image co-registration algorithm is proposed, which can eliminate the last term of (9) while performing phase correction, and sometimes it has higher accuracy than the 1-D range profile parameter estimation method proposed in [17]. After image co-registration, by carrying out FFT to $t_m$, the 2-D ISAR images can be yielded as

$$
S_\rho(r, f_d) = \sum_p \Theta_\rho \sin \left(T_{\text{obs}} f_d - \frac{2}{c} f_r v_p\right) \exp \left[-j\frac{4\pi}{c} f_r f_d\right]
$$

(10)

$$
S_\Gamma(r, f_d) = \sum_p \Theta_\rho \sin \left[T_{\text{obs}} f_d - \frac{2}{c} f_r v_p\right] \exp \left[-j\frac{4\pi}{c} f_r f_d\right]
$$

(11)

where $T_{\text{obs}}$ is the total imaging accumulation time, taking $A$ and $B$ as the instance, and the interferometric phase along the baseline $AB$ can be obtained by interfering with (10) and (11), which can be expressed as

$$
\theta_{\rho AB} = \angle \left(S_{\text{int}}(r, f_d) S_{\text{int}}(r, f_d)\right) = 2\pi f_r LB_{\rho AB}/c
$$

(12)

Therefore, the coordinate of the scatterer $p$ in the direction of the $U$-axis can be obtained by adopting the transformation relationship between the interference phase and its coordinate, which is described as

$$
U_p = \frac{\theta_{\rho AB} R_{\rho AB} c}{2\pi f_r L} + \frac{L}{2}
$$

(13)

In the far field, $B_{\rho AB} = (2U_p - L)/(2R_{\rho AB})$. Similarly, by performing the interference processing of the $AC$ baseline, the coordinate of scatterer $p$ on the $W$-axis can also be obtained. By performing the above processing on all the scatterers of the target and combining the distance measurement, the 3-D InISAR image could be generated consequently.

3 2-D high-resolution imaging through variational Bayesian compress sensing

In practical multifunction radar imaging system, less received echoes can be used for 2-D ISAR imaging in a single scan period, resulting in a shorter coherent processing interval of the target. It is difficult to obtain a satisfactory lateral resolution for azimuth compression directly to (8) and (9) during the short coherent processing interval so that the scatterers in the obtained ISAR images cannot be completely distinguished, which brings challenges for the 3-D InISAR imaging. At present, a large number of the CS algorithms are given to achieve the azimuth high-resolution of the target with limited pulses. Bayesian method is better by comparing with the other CS algorithms because of the introduction of the noise posterior information. In this section, the variational Bayesian interference algorithm is proposed to yield the super-resolution ISAR images for the target with less echoes. Compared with the Bayesian super-resolution method (SBL) in

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Assume that \( S_T \) and its probability density function is expressed as

\[
p(S_T | G_T, \alpha) = \prod_{n=1}^{N} \mathcal{N}(S_{T,n} | FG_{T,n}, \alpha^{-1}I)
\]

where \( S_{T,n} \) and \( G_{T,n} \) are the \( n \)-th column of \( S_T \) and \( G_T \), respectively. Assume that \( \alpha \) follows the Gamma distribution: \( p(\alpha | a, b) = \Gamma(\alpha, a, b) \), \( a \) and \( b \) are small constants. Here, we consider the ISAR images \( G_T \) are hierarchically modelled, without loss of generality, each column of \( G_T \) is assumed to be an independent and zero mean, complex Gaussian prior distribution.

\[
p(G_T | \beta) = \prod_{n=1}^{N} \mathcal{N}(G_{T,n} | 0, \eta_{\beta,n})
\]

where \( \eta_{\beta,n} = \text{diag}(\beta_{\text{var}1}, \beta_{\text{var}2}, \ldots, \beta_{\text{var}P}) \), \( \beta \) is a hyperparameter that obeys an independent gamma distribution and its probability density function is expressed as

\[
p(\beta | \mu_\beta) = \prod_{p=1}^{P} \Gamma(\beta_{\text{var}} | \xi, \lambda_\beta)
\]

where

\[
p(\lambda | c, d) = \prod_{n=1}^{N} \Gamma(\lambda_{\beta,n} | c, d)
\]

Since the parameter \( \lambda \) determines the sparseness of the ISAR images, we supposed it satisfies the Gamma distribution that is shown in (19). \( \xi, c, \) and \( d \) are all constants. By combining the Bayesian theorem and the independence of the parameters, the joint posterior distribution of the parameters is

\[
p(G_T, \lambda, \beta, \alpha | S_T, a, b, c, d) = \frac{p(S_T | G_T, \alpha)p(G_T | \beta)p(\lambda | c, d)p(\alpha | a, b)}{p(S_T)}
\]

Unfortunately, the distribution function \( p(S_T) \) is usually not available, making the calculation of (20) face certain challenges. [18, 19] proposed the variational Bayesian algorithm which is more effective by compared to the Markov chain Monte–Carlo sampling to solve (20). According to the unique assumption of the variational Bayesian inference, (20) can be approximated as

\[
p(G_T, \lambda, \beta, \alpha | S_T, a, b, c, d) \approx Q(G_T)Q(\lambda)Q(\beta)Q(\alpha)
\]

where \( Q(G_T) \), \( Q(\lambda) \), \( Q(\beta) \), and \( Q(\alpha) \) are posteriors of the unknown variables \( G_T, \lambda, \beta, \) and \( \alpha \), respectively, and then we can estimate them by the expectation-maximisation (EM) algorithm as follows [18]. The approximated posterior of \( G_T \) is described as (see (22))

\[
\mathbf{\mu}_n = a\Sigma_n F^H G_n
\]

\[
\Sigma_n = (aF^H F + \text{diag}(1/\beta_n))^{-1}
\]

From (22), it is easy to find that \( G_{T,n} \) obeys the complex Gaussian distribution with mean \( \mathbf{\mu}_n \) and variance \( \Sigma_n \). Keeping only the terms of \( Q(\lambda) \) that depend on \( \lambda \), then

\[
Q(\lambda) \propto \exp(\langle \log(p(\beta | \lambda))p(\lambda | c, d) \rangle_{Q(\beta)})
\]

\[
\propto \exp \left( \sum_{n=1}^{N} \sum_{p=1}^{P} \frac{\log(\eta_{\beta,n}/\Gamma(c))}{\beta_{\text{var}n}} \exp(-\lambda_{\beta,n}) \right)
\]

\[
\propto \exp \left( \sum_{n=1}^{N} \frac{1}{\beta_{\text{var}n}} \exp\left(-\sum_{n=1}^{N} (\beta_{\text{var}n} + d) \lambda_{\beta,n}\right) \right)
\]

As each component of \( \lambda \) is independent, then \( Q(\lambda_{\beta,n}) \) can be expressed as

\[
Q(\lambda_{\beta,n}) \propto \lambda_{\beta,n}^{P_{\text{var}} - 1} \exp\left(-\left(\sum_{p=1}^{P} \beta_{\text{var}n} + d\right) \lambda_{\beta,n}\right)
\]

Similarly, the posterior \( Q(\alpha) \) can be computed as (see (27)) where \( \| \cdot \|_F \) is Frobenius norm, and \( \text{tr}(X) \) is the trace of matrix \( X \). Next, the posterior \( Q(\beta) \) can be calculated as

\[
Q(\beta) \propto \sum_{n=1}^{N} \frac{1}{\beta_{\text{var}n}} \exp\left(-\sum_{n=1}^{N} (\beta_{\text{var}n} + d) \lambda_{\beta,n}\right)
\]
At this point, by maximising (26), (27), and (28), the estimated value of the parameters can be obtained as

\[
\lambda_n = \frac{P_n^2 + c}{\sum_{p=1}^P \beta_{pa}} + d_n
\]

(30)

\[
\alpha = \frac{N + a}{b + \| S_T - F G_T \|_2^2 + \sum_{n=1}^N \text{tr}(F_S F_S^T)\alpha} \tag{31}
\]

\[
\beta_{pa} = \sqrt{K_T}^{-1} \frac{k_{\beta}(2P_n)}{k_{\beta}^{-1}(2P_n)} \tag{32}
\]

where \( k_{\beta}(\cdot) \) is the modified second kind of Bessel function. The high-resolution ISAR images could be reconstructed through iteratively updating (23) (24) (30) (31) and (32) until the convergence is reached.

### 4 Simulation results

In this section, the effectiveness of the Variational Bayesian algorithm for 2-D high-resolution ISAR imaging with limited pulses and the application of the algorithm in 3-D InISAR imaging will be both verified through the simulation experiments.

It is supposed that the transmitter transmits the LFM signal with the following parameters: the carrier frequency \( f_0 \) is 10 GHz, the bandwidth \( B \) is 200 MHz, the pulse width \( r \) is 10 us, and the pulse repetition frequency is 625 Hz. The sampling frequency \( f_s \) in fast time domain is set to 25.6 MHz, which satisfies the requirement of the intermediate frequency complex sampling theorem. The distance between the ship and radar \( A \) is 8 km, and the length of the baselines \( AB \) and \( AC \) are both 2 m. Assume that the background noise is additive white Gaussian noise and the signal-to-noise ratio (SNR) in the signal domain is 15 dB. Fig. 2 is the ideal model of the ship target which is used for simulation in this paper. Consider it moves at a uniform velocity of 30 knots in the direction of 15 degrees with the \( U \)-axis in the UOV plane. Not only that, the ship also has complicated 3-D rotation in addition to the translational velocity due to the influence of the sea conditions, and the corresponding rotation parameters are listed in Table 1. It is assumed that the receivers only receive 64 echoes in short CPI.

Fig. 3 shows the 2-D ISAR imaging results for the target with limited pulses by different algorithms, where Fig. 3a is the ISAR image obtained by RD algorithm, Fig. 3b is the super-resolution ISAR image gained by using variational Bayesian algorithm. It is obvious that the azimuthal scatterers in Fig. 3a are not distinguished because of the short coherent processing interval, while the super-resolution image of each scatterer is obtained by adopting the variational Bayesian compress sensing in Fig. 3b, which verifies the effectiveness of the technique. Besides, it can be seen from Fig. 4 that the variational Bayesian can achieve better image quality than SBL algorithm because the image entropy of variational Bayesian is smaller than that of the SBL.

Figs. 5 and 6 are the 3-D InISAR images of the target, where (a) is the position of the scatterers in plane \( U-V \), (b) is the position of scatterers in plane \( U-W \), (c) is the position of the scatterers in plane \( V-W \), and

![Fig. 2 3-D ideal model of ship target](image)

**Table 1 3-D rotation parameters of the target**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Roll (deg)</th>
<th>Pitch (deg)</th>
<th>Yaw (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>4.8m/180</td>
<td>1.2m/180</td>
<td>4.5m/180</td>
</tr>
<tr>
<td>Value</td>
<td>2m/12.2</td>
<td>2m/6.7</td>
<td>2m/14.2</td>
</tr>
</tbody>
</table>
(d) is the position of the scatterers in UVW. From Fig. 5, we can see that the 3-D coordinates of all the scatterers on the target have been reconstructed with extremely high precision because the interference phase of each scatterer can be obtained after high resolution of Bayesian imaging. However, if we use RD algorithm to achieve 2-D ISAR image, and the interference phase of some scatterers cannot be obtained due to the coincidence of some scatterers in the imaging plane, resulting in the incomplete reconstruction for the 3-D InISAR image in Fig. 6.

5 Conclusion

This paper proposed a 3-D InISAR imaging technique for the target with limited pulses. It models each radar echoes after motion compensation and image co-registration as a sparse optimisation problem, and solves the problems by using the variational Bayesian

Fig. 5 3-D InISAR images of the target by using the variational Bayesian algorithm
(a) The position of the scatterers in plane U-V, (b) The position of the scatterers in V-W, (c) The position of the scatterers in plane U-W, (d) The position of the scatterers in plane UVW

to obtain super-resolution 2-D ISAR images. Then, the 3-D InISAR images of the targets with limited pulses can be reconstructed by combining the interference processing and range measurement. The comparison results in simulation experiments illustrate the validity for the novel technique in this paper.

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7 References


