GPS/INS integration based on adaptive interacting multiple model

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Abstract: The extended Kalman filter (EKF) had widely been used in the inertial navigation system (INS) and global positioning system (GPS) integrated navigation system. Nevertheless, the EKF contains instability caused by linearisation and numerous calculations of the Jacobian matrix. To solve the problem, the adaptive interacting multiple model (AIMM) filter algorithm is proposed in order to achieve a better navigation solution. The soft-switching feature, which is offered by interacting multiple model, allows conversion of process noise between lower and upper limits, meanwhile the measurement covariance is adjusted online by Sage adaptive filter. Considering the need to update the pseudo-range and Doppler data, the updating strategy of classification measurement is proposed. The results of the GPS/INS integrated navigation are estimated in the form of data of real ship, and experimental results indicate that the higher position accuracy can be obtained in AIMM filter.

1 Introduction

The global positioning system (GPS)/inertial navigation system (INS) integrated navigation system can be consisted of differential GPS (DGPS) to get better positioning precision [1]. Generally speaking, if GPS is used to correct the cumulative deviations including position and velocity of INS, INS offers higher accuracy solution when the GPS signal is defective. The credible solutions will be given by GPS on condition that satellites exceed four [2]. So it is a prominent shortage when GPS works independently. In normal case, three ideas are utilised to solve this problem, high precision INS, other accessory equipment, and advanced technologies and algorithms. The latter two ideas are applied to achieve the high precision navigation solution in this paper.

As a common data fusion method, extended Kalman filter (EKF) [3] has been widely utilised in various fields [4, 5]. The non-linear observation and state models would be linearised on the basis of the first-order Taylor series expansion on the condition that the noise is deemed as Gaussian. However, EKF may not be able to generate effective solution estimates, especially when GPS is interrupted, because higher order terms have been neglected in the process of calculation. This is extremely common if low-cost MEMS-based inertial measurement units (IMU) is applied in GPS/INS integrated navigation system. Then, the unscented Kalman filter (UKF) [6] was proposed as a kind of linear regression estimated filtering algorithm. In order to acquire the posteriori mean and covariance accurately, UKF multiplies a set of deterministic sampling points, which offer suitable weights for any non-linear problem through the non-linear measurement equations and dynamic equations. There is a very difficult problem that weak points of Gaussian distribution hypothesis should be solved [7]. Nevertheless, the performance of the enhanced KFs mainly relies on the system. The precision of mode evaluation can be poor when the system shows strong non-linear. During the GPS signal is defective, the navigation solution, which provided by KF, should be diverged in accordance with process of linearisation system model [8]. To cope with the weakness of KF-based algorithms, a large number of suitable endeavors have been provided [9]. A commonly used method of on-line adjustment of covariance matrix, named innovation adaptive estimation, which would be applied to coping with the instability of KF-based algorithms. Innovation adaptive estimation Kalman filter [10] can be proposed in order to impeding the effective of ambient noise and reducing filter divergence. In addition, residual sequences based on innovation adaptive estimation Kalman filter can be used to heighten the randomness features of on-line filter. In order to ensure the better estimation accuracy, an adaptive filtering algorithm based on memory attenuated filter is further researched. As an alternative method, a kind of multi-model (MM) estimation method had been investigated. Among massive MM estimation methods, the interacting multiple model (IMM) filtering, as the most powerful state estimation method, has been utilised in the field of multi-sensor data fusion [11]. Furthermore, this method has the ability to evaluate the dynamic system state variables with substantial behaviour models, which are deemed as a probabilistic switching method. Yet, its validity is seldom verified in practical tests. An enhanced adaptive interacting multiple model (AIMM) filter approach is proposed to verify the validity in practical sea trials in this thesis.

This paper is organised as follows. First, the complete mathematical model of the Strapdown Inertial Navigation System (SINS)/GPS integrated system is described. Second, the AIMM filter algorithm is introduced to enhance the capability of GPS/INS integrated system, and then the influence of different algorithms is compared and analysed through practical sea trials. Last, main conclusions are given.

2 SINS/GPS integrated system model

2.1 State equation

The psi-angel deviation models of INS should be represented as

\[
\begin{align*}
\delta p &= -\phi_w \times \delta p + \delta s \\
\delta s &= -2\phi_w \times \phi_w \times \delta s - \delta \theta \times \tau + \zeta \\
\delta \theta &= - (\phi_w + \phi_w) \times \delta \theta + \sigma
\end{align*}
\]

(1)

where \(\delta p, \delta s\) and \(\delta \theta\) denote the heading, velocity and position error vectors, respectively. \(\phi_w\) represents the rate of earth relative to inertial frame. \(\phi_w\) represents the rate of navigation frame relative to earth. \(\tau\) denotes the specific force vector. The expansions that include the accelerometer bias error vector \(\zeta\) and the gyro drift error vector \(\sigma\), can provide the systematical error of GPS/INS integrated navigation system.
The error states model in INS/GPS integrated navigation system should be represented as

\[
\begin{align*}
X_{\text{NAV}} &= [\delta P_N, \delta P_e, \delta P_d, \delta V_N, \delta V_e, \delta V_d, \delta h_N, \delta h_e, \delta h_d]^T \\
X_{\text{Acc}} &= [\delta h_N, \delta h_e, \delta h_d, \delta f_N, \delta f_e, \delta f_d]^T \\
X_{\text{Gyro}} &= [\sigma_{hN}, \sigma_{hE}, \sigma_{hD}, \sigma_{fN}, \sigma_{fE}, \sigma_{fD}]^T \\
X_{\text{Act}} &= [\delta l_{hN}, \delta l_{hE}, \delta l_{hD}]^T
\end{align*}
\]  

where \(\delta P_N, \delta P_e, \delta P_d\) indicate position errors in east, north and down orientations, respectively. \(\delta V_N, \delta V_e, \delta V_d\) indicate velocity errors in east, north and down directions, respectively. \(\delta h_N, \delta h_E, \delta h_D\) indicate direction error in east, north and down directions, respectively. \(\delta f_N, \delta f_E, \delta f_D\) indicate accelerometer error vectors, and \(\sigma_{hN}, \sigma_{hE}, \sigma_{hD}, \sigma_{fN}, \sigma_{fE}, \sigma_{fD}\) indicate gyro drift errors vector. \(\delta l_{hN}, \delta l_{hE}, \delta l_{hD}\) indicate lever arm errors.

The lever arm errors are defined as stochastic constants, and the inertial sensor errors are modeled as first-order Gauss–Markov processes. With regard to the low-cost inertial sensors, the inertial sensor errors will evidently reduce the navigation performance in short term until combined with other sensors in order to revise the navigation result.

### 2.2 Measurement equation

According to the relationship between the measured value derived from INS and the original GPS observation value, the measurement state vector on the basis of KF is obtained. The GPS observation value may be mainly collected by Doppler, carrier phase, and pseudo-range method. To achieve higher precision results, the carrier phase measurement values should be applied in the step of filtering update. Based on the actual situation, this thesis will use the dual-differential pseudo-range information and single-differential Doppler information in order to achieve higher precision in integrated navigation system. The GPS measurement equation can be followed as

\[
\begin{align*}
\Delta V_{\text{GPS}} - \Delta V_{\text{INS}} &= H^c C^e \phi + H^c C^e L^b + \Delta V e_p \\
D V_{\text{GPS}} - D V_{\text{INS}} &= \frac{1}{\lambda_{\text{GPS}}} H^c C^e \phi + \Delta V e_D
\end{align*}
\]  

where \(\Delta V\) represents differential numerical information between the two contiguous satellites, and \(\Delta V\) stands for the double differential value. \(\Delta V_{\text{GPS}}\) and \(D V_{\text{GPS}}\) observe numerical observation value of pseudo-range and Doppler provided by GPS, respectively; \(\rho_{\text{INS}}\) and \(D \rho_{\text{INS}}\) refer to the pseudo-range and Doppler information from INS, respectively. \(\phi\) and \(e_D\) refer to the pseudo-range and Doppler information about measuring noises, respectively. \(H^c\) indicates the geometry transform matrix; \(C^e\) and \(C^e\) indicate rotation matrix which are from body frame to earth frame and from navigation frame to earth frame, respectively. \(\lambda_{\text{GPS}}\) indicates a signal wavelength.

The error between GPS antenna and IMU centre must be discussed according to the non-determinacy of lever arm, particularly the IMU estimated values should be updated. The IMU position can be written as

\[
r_{\text{GPS}} = r_{\text{INS}} + C_l^b L^b
\]  

where \(L^b\) indicates the deviation vector of lever arm.

Similarly, the relation between the GPS antenna and the IMU velocity is

\[
v_{\text{GPS}} = C_l^e v_{\text{INS}} + C_l^e (\Omega^b - \Omega^e) L^b
\]  

### 2.3 SINS/GPS fusion based on EKF

An EKF can be utilised in this thesis for the sake of integrate the result of inertial equipment with GPS measuring values. The analytic result of EKF is recursively procedure, which includes forecast process that is determined by

\[
\begin{align*}
x_m &= x_{m-1} + \Phi_m x_{m-1} \\
P_m &= \Phi_m P_{m-1} \Phi_m^T + Q_m + \Omega_{m-1}
\end{align*}
\]  

where \(\Phi_m\) and \(x_{m-1}\) indicate prior state vector at \(m\) and posterior state vector at \(m-1\), respectively. \(P_{m-1}\) and \(P_m\) denote the prior covariance matrix of \(x_m\) and the posterior covariance matrix of \(x_{m-1}\), respectively. \(Q_m\) stands for the covariance matrix of process noise. The correction step can be written as

\[
\begin{align*}
G_m &= P_m D_m^H (H_m P_m D_m^H + C_m)^{-1} \\
x_m &= x_{m-1} + G_m (R_m - D_m x_{m-1}) \\
P_m &= (I - G_m D_m) P_m
\end{align*}
\]  

where \(G_m\) indicates the gain of Kalman. \(R_m\) and \(D_m\) represent the residual vector and residual matrix, respectively. \(C_m\) indicates the covariance matrix which comes from measurement noise, and then \(x_m\) denotes posterior state vector at time \(m\).

If the GPS measuring values are an available option, the evaluated parameters can be updated and initial inertial measuring values can be corrected by using the estimated sensor errors.

If the suitable statistical characteristics in which the process noise and measurement noise are described, Kalman filter would often achieve an accurate results. On condition that the mathematical equations inaccuracy, the convincing solutions are given by both KF-based derivation methods and Kalman filter.

### 3 System architecture

#### 3.1 Interacting multiple model filtering

The IMM filter has been widely concerned because of their lower computational cost and higher performance. So the IMM estimates have been applied in achieving the state estimates based on the probability model. The IMM filtering algorithm can be contained as follows

#### 3.1.1 Interaction and mixing: First of all, the integrated navigation model may be deemed as a set of discrete \(r\) models which would be described models as \(M = \{M_1, M_2, \ldots, M_r\}\). Hence, mixing probability \(\mu_{ij}\) can be described as

\[
\mu_{ij} = \frac{1}{c_i} p_i \mu_{mi-1}
\]  

The normalisation factor \(c_i\) is given as

\[
c_i = \sum_{j=1}^{r} p_j \mu_{mj-1}
\]  

where \(p_j\) denotes the mode switching probability matrix and \(\mu_{mj-1}\) denotes the mode probability of model \(i\). The initial mixed state estimate \(x_{m-1}^{0}\), and its covariance \(P_{m-1}^{0}\) for each filter can be expressed as follows:

\[
x_{m-1}^{0} = \sum_{i=1}^{r} p_i x_{m-1}^{0+i}
\]  

\[
P_{m-1}^{0} = \sum_{i=1}^{r} p_i P_{m-1}^{0+i}
\]
where \( P_{m}^{j} \) and \( x_{m}^{j} \) indicate the ultimate covariance and mean value based on single model \( i \), respectively.

3.1.2 Mode probability update: The likelihood value based on KF forecast and update for every filter can be achieved if the initial mixed state estimation and covariance of preliminary procedure is executed. In addition, the state mean and covariance are also evaluated subsequently. Hence, the likelihood function of every filter can be simplified as

\[
\Lambda_{m} = \frac{1}{(2\pi)^{n/2}S_m^{1/2}} \exp\left(-\frac{1}{2}(v_m - \bar{S}_m)^T(\bar{S}_m)^{-1}(v_m - \bar{S}_m)/2\right)
\]

where \( n \) denotes the measuring frequency, \( v_m \) and \( S_m \) indicate the innovation sequence and its associated covariance matrix, respectively, and then can be written as

\[
v_m = z_m - H_m \bar{x}_m
\]

\[
S_m = H_m P_m H_m^T + R_m
\]

The renewed probability model of every filter will be calculated as

\[
\mu_{m}^{i} = \frac{1}{c} \Lambda_{m} \phi_{i}
\]

where \( c \) denotes normalisation coefficients of a mode probability update, and can be calculated as

\[
c = \sum_{i=1}^{m} \Lambda_{m} \phi_{i}
\]

3.1.3 Combination: During this step, the prediction will be executed due to the renewed probability model of every filter. The state estimation \( x_m \) and covariance \( P_m \) can be written as

\[
x_m = \sum_{i=1}^{m} \mu_{m}^{i} x_{m}^{i}
\]

\[
P_m = \sum_{i=1}^{m} \mu_{m}^{i} [P_{m}^{i} + (x_{m}^{i} - x_m)(x_{m}^{i} - x_m)^T]
\]

3.2 Adaptive Kalman filtering

During the process of actual GPS/INS integrated navigation system, the numerical value of measured noise will vary with time. To improve the positioning precision of GPS/INS integrated navigation system, statistical values which stem from measured data, the method of updating is implemented by considering measurements. Each filter should execute the forecast and update phases when GPS observed value and INS observed value are known. The mode probability can be updated on condition that the measurement update stage has been performed. By updating the status of dual-model IMM filter, a weighted set can be computed properly. The final navigation solutions are obtained in accordance with mixed formula, and then the residual results can be used to update the measuring noise covariance. Finally, the error states vectors of various sensor, which have been evaluated in IMM filtering algorithm, have been fed back to INS mechanisation stage in order to offsetting inertial initial outputs.

4 Sea trials and data analysis of results

4.1 Description of sea test

A sea test had been implemented around the Dalian harbour, China. The influences on navigation results is analysed and discussed by using an ordinary navigation grade IMU and one high-grade GPS receivers in detail. The whole sailing time of sea trial was about 21 h, the ship is in continuous sailing and the distance travelled by a sailing vessel is nearly 200 n mile. Only about 50 min (red circle) were chosen as the research target.

For the purpose of implementing the influence evaluation of the GPS/INS integrated navigation system, only one frequency range and Doppler measurements given by GPS have utilised. The reference solution is obtained through combine IMU measured value with DGPS carrier phase measured value.

4.2 Influence evaluation and comparison of AIMM algorithm

In order to better discuss the influence of the AIMM algorithm, the validity of AIMM algorithm is analysed through low-cost INS is combined with GPS. We investigate a comparison among the EKF, IMM and AIMM filtering information fusion technology in light of the ship integrated navigation system.
The prior altitude correlation weighted algorithm is expressed using the concrete elevation \( e \) which stem from non-differential measuring covariance

\[
 r_{ii} = \sigma_0^2 / (\sin(e))^2 \quad (22)
\]

where \( \sigma_0 \) denotes the standard deviation. In this paper, pseudo-ranges is 0.4 m and Doppler observed value is 0.02 m/s. Then, a kind of adaptive algorithm based on IMM filter evaluates the online measuring covariance, which is utilised to improve influence and performance of GPS/INS integrated navigation system. The pseudo-range and Doppler observation values can be used to update the KF.

General speaking, creating a more accurate stochastic model related to INS, which is better implementation in various situations, presents an obvious challenge. To addressing this problem, this research both proposes a kind of dual-model IMM filtering algorithm and sets two covariance matrices of process noise, \( Q_l = 4Q_0 \) and \( Q_s = 0.25Q_0 \). Where \( Q_0 \) is nominal covariance, which can be decided based on the specific parameters of sensor factor.

The dual-model IMM filtering algorithm had been applied on condition that the pre-supposition values had been determined using empirical formula. The probabilistic values should be expressed by a Markov transition matrix

\[
 [p_t] = \begin{bmatrix}
 0.98 & 0.02 \\
 0.02 & 0.98 
\end{bmatrix} \quad (23)
\]

Actually, the probabilistic values have limited influence on the end results. To improve the stability of filter at the beginning of the research, it is necessary to set some values of original model probability. So 0.4 and 0.6 are defined as the original model probabilities which correspond to smaller and large process noise, respectively.

Fig. 3 indicates position errors of three different orientations on the basis of GPS/INS integrated navigation system. EKF and IMM filter can be applied to compare the influence of AIMM filter. For position precision, the adaptive IMM filter algorithm provides optimal solutions comparing with two other approaches. Owing to the inaccurate statistical features including the dynamic equation and the measurement equation, the solutions of EKF contain numerous noises.

Despite a continuous smooth path for integrated navigation system can be displayed, few position errors are decreased under EKF algorithm. Yet, the ordinary IMM algorithm provides lower positioning accuracy.

Fig. 4 denotes the positioning RMSE of three different filters. Compared with EKF algorithm, the AIMM filter algorithm improves the positioning results of GPS/INS integrated system by 35.3%, 37% and 31.6% in the three different orientations. Fig. 5 indicates the speed and its errors using AIMM filter algorithm. It would be shown that AIMM filter algorithm achieves centimeter level precision in three different kinds of directions. Furthermore, it generates a deviation of 0.27 m/s which is due to the vessel steering.
and 47.3%, respectively, compared to EKF filter. However, there is a slight drop in down component.

Fig. 7 denotes the performances of attitude and attitude errors on the basis of AIMM filter algorithm. It can be concluded that the precision of roll and pitch are evidently superior to heading. Fig. 8 illustrates the AIMM filter algorithm has better performance in attitude estimation than EKF. The attitude precision of IMM filter algorithm has identical with AIMM filter algorithm basically. Filter switching capability is checked by model probability of AIMM. The capability of soft-switching ensures that the filter can switch independently between lower and upper bounds of $Q_k$.

5 Conclusion

In this paper, a kind of AIMM filter algorithm for ship is proposed. The presented algorithms are tested with sea trial in Dalian port, and the results demonstrate that AIMM filter yields the best accuracy under the same condition. The estimated precision was improved as the covariance between process and measurement noises should be independently adjusted by AIMM filter algorithm. Moreover, considering the computation cost needs to be decreased, a specific sequential update approach should be introduced, and the measuring covariance should also be used on the basis of on-line Sage adaptive filtering. The results of the GPS/INS integrated navigation are estimated according to experiment of real ship, and experimental results indicate that the higher position accuracy can be obtained in AIMM filter.

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7 References