Coherent side-band BOC processing

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Abstract: Binary offset carrier (BOC) modulated signals are characterised by multi-peaked correlation functions which may lead to false peak locks that occur when a secondary correlation lobe is tracked instead of the main peak. To solve this problem, side-band processing (SBP) operates on the two main lobes of the BOC spectrum separately, in a binary phase shift keying like fashion. Although an unambiguous correlation function is obtained, the information provided by the BOC subcarrier is lost. This information is preserved by the double estimator and by the double phase estimator (DPE) which adopt an additional tracking loop to process the subcarrier component. In this study, the connection between SBP and independent subcarrier processing is shown and a novel SBP scheme, the coherent side-band (CSB) approach, is proposed. As in standard SBP, the two main lobes of the BOC spectrum are processed separately and upper and lower correlators are computed. These correlators are then combined to recover the subcarrier information. CSB provides subcarrier measurements and achieves the performance of the DPE, CSB tracking is thoroughly characterised and simulations and real-data processing are used to support theoretical findings. The CSB approach is an effective alternative to independent subcarrier processing.

1 Introduction

Modern global navigation satellite system (GNSS) signals adopt a subcarrier to shape the signal spectrum, increase frequency separation between different signals and improve tracking performance. Binary offset carrier (BOC) modulated signals are characterised by a rectangular subcarrier which split the signal spectrum around two main lobes at $\pm f_{sub}$, the subcarrier repetition frequency [1]. Although BOC subcarriers lead to sharp correlation functions, they also introduce secondary peaks which may lead to false secondary peak lock: the tracking loop used to process the BOC signal may lock into a secondary peak introducing biases in the parameters tracked.

To solve this problem, several unambiguous BOC tracking techniques have been introduced. These techniques include bump jump (BJ) [2], autocorrelation side-peak cancellation technique (ASPeCT) [3] and its extensions [4], side-band processing (SBP) [5–7], independent subcarrier processing with the double estimator (DE) [8, 9] and the double phase estimator (DPE) [10, 11], pre-filtering and local subcarrier shaping [12] and subcarrier cancellation (SCC) [13, 14]. A schematic representation of the different unambiguous BOC tracking techniques and of their relationship is shown in Fig. 1. SBP techniques are represented in the first column of Fig. 1: the SBP working principle exploits the fact that the BOC spectrum can be approximated as the sum of two frequency-translated binary phase shift keying (BPSK) spectra. Single side-band (SSB) processing is obtained when only one side of BOC spectrum is considered [6, 7]. The two BOC main lobes can also be processed separately and then non-coherent combining can be performed at the output of the tracking loop discriminator [5–7]. Although SBP leads to unambiguous correlation functions, the information provided by the BOC subcarrier is lost. The subcarrier benefits are also lost with techniques such as BJ [2], ASPeCT [3, 4], pre-filtering [12] and SCC [13, 14].

To fully exploit the benefits of the subcarrier, it is necessary to consider it in a similar way as the other signal components, the code and the carrier. The signal code is used to provide pseudoranges whereas the carrier generates Doppler measurements and carrier phase observations. The subcarrier should be processed with a dedicated tracking loop and used to generate subcarrier measurements. This principle is exploited by the DE [8, 9] and the DPE [10, 11] which adopt an additional tracking loop to process in an independent way the subcarrier component.

This paper describes a new SBP scheme which coherently combines the correlators computed using the two side lobes of the BOC spectrum. The approach proposed is denoted as coherent side-band (CSB) processing and is equivalent to the DPE. In particular, the upper and lower correlators, computed from the BOC main spectral lobes, can be recombined to form the prompt and the orthogonal prompt correlators used by the DPE. This shows the connection between subcarrier independent processing and SBP. The upper and lower correlators are also used to recover subcarrier information. In particular, the CSB approach allows one to obtain subcarrier measurements without requiring correlation with different subcarrier replicas. In the CSB approach, carrier and subcarrier components are strictly related and are processed jointly for the generation of the upper and lower correlators. Carrier and subcarrier information is extracted using a dual-phase discriminator which has been designed using the maximum likelihood (ML) approach. Discriminator outputs are filtered and combined to generate the local carriers for the down-conversion of the upper-and lower-band components of the BOC signal.

CSB tracking is thoroughly characterised and simulations and real-data processing are used to support theoretical findings. In particular, the coherent output signal-to-noise ratio (SNR), the CSB subcarrier tracking jitter and the subcarrier multipath error envelope (SMEE) are theoretically analysed. The CSB approach achieves the same coherent output SNR of the DPE. Moreover, when using ML discriminators, the tracking jitter of the subcarrier component is very close to that of the DPE. Theoretical results are supported by semi-analytic simulations and by real-data processing. A good agreement between theoretical and simulation results is obtained. Real sine BOC(1,1) and cosine BOC(15, 2.5) signals collected from the Galileo In-Orbit Validation Element (GIOVE)-B satellite are used to test the effectiveness of the CSB approach. The experimental results further support the theoretical findings and show that CSB processing achieves the same performance of the DPE.

The analysis performed shows that the CSB approach is an effective alternative to independent subcarrier processing.
particular, the same performance of the DPE can be achieved with the flexibility provided by SBP schemes. More specifically, different processing schemes can be implemented and computational reduction can be achieved using a down-sampling estimate the signal parameters. This is a notational choice which does not impact the derivations provided in the following. Finally, a GNSS receiver usually recovers several signals from different satellites. However, each signal is characterised by a specific code and the receiver is able to process each signal independently, exploiting the quasi-orthogonality of the codes. For this reason, a single useful component is considered in (1).

Signal (1) is filtered and down-converted by the receiver front-end before being digitised. In the following, the effects of quantisation are neglected and it is assumed that the signal is sampled without introducing significant distortions. After down-conversion and sampling, (1) becomes

\[
y[n] = \sqrt{C d(nT_s - \tau_0) \tilde{c}(nT_s - \tau_0) d_0(nT_s - \tau_0) e^{j2\pi f_0 T_s + j\phi_0}} + \eta[n]
\]

where the notation \(x[n]\) is used to denote a discrete time sequence sampled at the frequency \(f_s = (1/T_s)\). The index ‘BB’ is used to denote a signal down-converted to base-band and the symbol \(\tilde{\cdot}\) is used to indicate the impact of the front-end filter on the useful signal components.

The noise term, \(\eta[n]\), is assumed to be a complex additive white Gaussian noise (AWGN) with independent identically distributed (i.i.d.) real and imaginary parts each with variance \(\sigma^2\). This variance depends on the filtering, down-conversion and sampling strategy applied by the receiver front-end and is given by \(\sigma^2 = N_0 B_{RRx}\), where \(B_{RRx}\) is the front-end one-sided bandwidth and \(N_0\) is the power spectral density (PSD) of the input noise, \(\eta(t)\).

Note that front-end filtering may introduce correlation among noise samples. In this case, the hypothesis of a white input noise sequence is not strictly valid and correlation among noise samples should be accounted for [15, 16]. The front-end bandwidth, \(B_{RRx}\), is however much wider than that of the correlator blocks used to process the input signal. The correlation among noise sample can thus be neglected and the AWGN hypothesis adopted. The ratio between the carrier power, \(C\), and the noise PSD, \(N_0\), defines the carrier-to-noise density power ratio \((C/N_0)\), one of the main signal quality indicators used in GNSS.

In [10], the subcarrier component was approximated as a pure sinusoid. Using this approximation and when a sine-phased BOC is considered the subcarrier can be expressed as in (see (3)) where \(f_{sb}\) is the subcarrier frequency, \(\phi_{0}\) is the subcarrier phase and \(s_1\) is a scaling factor which depends on the filtering

\[
\tilde{s}_s(nT_s - \tau_0) \approx s_1 \sin(2\pi f_{sb} nT_s + \phi_0)
\]

\[
= s_1 \cos(2\pi f_{sb} nT_s - \pi/2)
\]

\[
= \frac{s_1}{2} \exp \{j2\pi f_{sb} nT_s + j\phi_{0}\} + \exp \{-j2\pi f_{sb} nT_s - j\phi_{0}\}
\]

Fig. 1 Schematic overview of the different BOC tracking techniques and of their relationships
applied to the received signal. This approximation implies that BOC signal (2) can be interpreted as the sum of two frequency-shifted BPSK signals

\[
y[n] = \sqrt{C_S} \left[ d(nT_s - \tau_0) \bar{c}(nT_s - \tau_0) \cdot \exp\{j2\pi (f_d + f_{\phi_0})nT_s + j(\phi_c + \phi_{\phi_0})\} \right] + \sqrt{C_S} \left[ d(nT_s - \tau_0) \bar{c}(nT_s - \tau_0) \cdot \exp\{j2\pi (f_d - f_{\phi_0})nT_s + j(\phi_c - \phi_{\phi_0})\} + \eta_{\text{BA}}[n] \right].
\]

Signal representation (4) is exploited for SBP of GNSS signals [6, 7]: the two side-band components of (4) are correlated independently as BPSK signals. Standard SBP techniques remove the subcarrier through non-coherent processing. In this paper, a coherent approach is adopted to recover the information provided by the subcarrier.

### 3 Coherent side-band processing

When BOC SBP is applied, the input signal, \(y[n]\), is correlated with two local signal replicas

\[
x_s[n] = c(nT_s - \tau) \exp\{j2\pi (f_d + f_{\phi_0})nT_s + j(\phi_c + \phi_{\phi_0})\} \quad (5)
\]

\[
x_s[n] = c(nT_s - \tau) \exp\{j2\pi (f_d - f_{\phi_0})nT_s + j(\phi_c - \phi_{\phi_0})\} \quad (6)
\]

where \(\tau\), \(f_d\) and \(f_{\phi_0}\) are the delay, the Doppler and subcarrier frequencies tested by the receiver. \(\phi_c\) and \(\phi_{\phi_0}\) are the carrier and subcarrier phases estimated by the receiver.

As a result, the upper and lower prompt correlators are obtained

\[
P^+ = \frac{1}{N} \sum_{n=0}^{N-1} y[n] x_s[n]^* = \sqrt{C_S} \frac{d}{\sqrt{2}} \frac{d\bar{R}(\tau - \tau_0)}{N} \sin\left(\frac{\pi}{\Delta f + \Delta f_{\phi_0}} nT_s\right) \cdot \exp\{j\pi \Delta f + \Delta f_{\phi_0} nT_s\} \cdot c(nT_s - \tau_0) \bar{c}(nT_s - \tau_0) \cdot \exp\{j\pi \Delta f + \Delta f_{\phi_0} nT_s\} \cdot \eta_s + \eta_c
\]

\[
P^- = \frac{1}{N} \sum_{n=0}^{N-1} y[n] x_s[n]^* = \sqrt{C_S} \frac{d}{\sqrt{2}} \frac{d\bar{R}(\tau - \tau_0)}{N} \sin\left(\frac{\pi}{\Delta f - \Delta f_{\phi_0}} nT_s\right) \cdot \exp\{j\pi \Delta f - \Delta f_{\phi_0} nT_s\} \cdot c(nT_s - \tau_0) \bar{c}(nT_s - \tau_0) \cdot \exp\{j\pi \Delta f - \Delta f_{\phi_0} nT_s\} \cdot \eta_s + \eta_c
\]

where

- \(\cdot^*\) denotes complex conjugation;
- \(d\) is the navigation bit assumed to be constant during the integration time;
- \(\bar{R}(\cdot)\) is the correlation between received and locally generated BPSK codes;
- \(\Delta f = f_d - f_s\) and \(\Delta \phi = \phi_c - \phi\) are the residual carrier frequency and carrier phase;
- \(\Delta f_{\phi_0} = f_{\phi_0} - f_{\phi_0}\) and \(\Delta \phi_{\phi_0} = \phi_{\phi_0} - \phi_{\phi_0}\) are the residual subcarrier frequency and subcarrier phase;
- \(\eta_s\) and \(\eta_c\) are two independent circularly symmetric Gaussian random variables with zero mean and total variance \(\sigma_r^2\);

\(N\) is the number of samples used for correlating the carriers and

\(T_s = NT_s\)

is the coherent integration time. The independence of the two noise components in (7) and (8) is due to the fact that they are obtained by projecting the input noise, \(\eta_{\text{BA}}[n]\), on two orthogonal waves, \(s_s[n]\) and \(s_r[n]\). The variance, \(\sigma_r^2\), depends on the power of the input noise, \(\eta_{\text{BA}}[n]\), and, under the hypothesis of independent input samples, it is given by

\[
\sigma_r^2 = \frac{1}{N} \sigma_n^2.
\]

In SSB processing only one of two correlators is used and the tracking loop adopted operates as for BPSK signals. In non-coherent side-band processing, \(P^+\) and \(P^-\) are used separately: two discriminators are used to extract the carrier information from the two signal side-bands. The information is then recombined in a non-coherent fashion neglecting or removing the subcarrier component.

It is noted that \(P^+\) and \(P^-\) contains the same information extracted by the DPE. In the DPE, two prompt correlators are obtained considering sine and cosine local subcarriers. When a sine BOC signal is considered, the prompt correlator, \(P_0\), computed by the DPE can be obtained from \(P^+\) and \(P^-\) as

\[
P_0 = (P^+ + P^-) \quad (11)
\]

whereas the orthogonal prompt correlator, \(P_o\), is given by

\[
P_o = (P^+ - P^-) \quad (12)
\]

Equations (11) and (12) directly derive from the linearity of the operations performed in (7) and (8) and by the definition of the local signal replicas, (5) and (6). In this respect, DPE and CSB processing are equivalent.

In this paper, a joint discriminator is designed to estimate the residual carrier and subcarrier phases, \(\Delta \phi\) and \(\Delta \phi_{\phi_0}\). These estimated residual phase errors will be used by the CSB tracking loop to align carrier and subcarrier components. More specifically, residual carrier and subcarrier errors will be driven to zero by the CSB loop. This process is shown in Fig. 2 which provides a schematic representation of the CSB tracking loop: carrier and subcarrier phase information is extracted by a dual-phase discriminator which combines \(P^+\) and \(P^-\). The discriminator outputs are then filtered using two standard loop filters [17]. In particular, \(F(z)\) and \(F_{\phi_0}(z)\) in Fig. 2 denote the transfer functions of the filters used in the carrier and subcarrier branches of the CSB loop.

The outputs of the filters are estimates of the carrier and subcarrier Doppler frequencies, \(f_d\) and \(f_{\phi_0}\) and thus can be used for the generation of carrier and subcarrier measurements. Moreover, \(f_d\) and \(f_{\phi_0}\) are combined and used to generate the exponential terms in local signal replicas (5) and (6). Local signal carriers are generated by the two numerically controlled oscillators (NCOs) depicted in Fig. 2. This operation closes the CSB processing loop.

#### 3.1 Dual-phase discriminators

Several discriminators can be used to determine the residual carrier and subcarrier errors, including the discriminators adopted by the DPE. In this paper, the following procedure is used. For the discriminator design, residual frequency errors, \(\Delta f\) and \(\Delta f_{\phi_0}\), are assumed to be zero. Under this assumption

\[
P^+ = \sqrt{C_S} \frac{d}{\sqrt{2}} \frac{d\bar{R}(\tau - \tau_0)}{N} \exp\{j\pi \Delta f + \Delta f_{\phi_0} nT_s\} + \eta_s
\]

\[
P^- = \sqrt{C_S} \frac{d}{\sqrt{2}} \frac{d\bar{R}(\tau - \tau_0)}{N} \exp\{j\pi \Delta f - \Delta f_{\phi_0} nT_s\} + \eta_c
\]

The ML approach [18] is then used to estimate \(\Delta \phi\) and \(\Delta \phi_{\phi_0}\). In particular, in Appendix 1 it is shown that the ML estimates of these two parameters are given by
\[ \Delta \phi = \frac{1}{2} \Delta \phi_p \]
where function \( \angle \cdot \) evaluates the angle of its complex input. It is noted that discriminators \((14)\) are insensitive to the navigation bits, \( d \), which are removed through the product between correlators. Equation \((14)\) defines the outputs of the dual-phase discriminator used by the CSB loop.

### 3.2 Delay lock loop (DLL) structure

To recover the signal delay, \( \tau_0 \), a DLL can be adopted. A standard DLL uses early and late correlators to maximise the correlation function, \( R(t - \tau_0) \), or equivalently to maximise the amplitude of the prompt correlator. In the CSB, the DLL has to be designed in order to maximise the composite correlator defined by \((11)\). In particular, \( P \) is given by

\[ P = \sqrt{c_s \exp(\Delta \phi \cos(\Delta \phi_b) + (\eta_+ + \eta_-))} \]  

When the subcarrier component is recovered by the CSB, the cosine term is approximately equal to 1. In this case, \((15)\) degenerates to the prompt correlator model of a BPSK signal, where the subcarrier component is absent.

In the CSB processing, a single DLL can be thus implemented using the following approach: upper and lower early and late correlators can be computed similarly to \( P_+ \) and \( P_- \). Then, composite early and late correlators can be obtained. Finally, the composite early and late correlators can be used in a standard DLL architecture \([19]\). In particular, standard DLL discriminator functions can be used.

Several other possibilities are available where \( E_+ \), \( E_- \), \( L_+ \) and \( L_- \), the upper and lower early and late correlators, are combined to form different error functions driving the CSB DLL. For example, non-coherent combining can be adopted. In this respect, the CSB provides a very flexible architecture allowing different forms of processing.

### 3.3 Algorithm extensions

Several strategies can be adopted to optimise the CSB approach and reduce computational requirements. In particular, the same approaches adopted for standard SBP can be used to reduce computational complexity \([5, 20]\).

A possible strategy to reduce the computational complexity of the CSB is shown in Fig. 3: the input signal, \( y[n] \), is split on two branches and multiplied by the nominal frequencies of the upper and lower side-band components. In this way, upper and lower signal components are down-converted around to the zero-frequency as depicted in Fig. 3b. When brought to base-band, the two components can be filtered and down-sampled as shown in Fig. 3a, producing two data streams that can then be fed to the two branches of the CSB loop in Fig. 2.

Down-sampling can significantly reduce the computational load associated to the evaluation of the correlators. Moreover, the processing shown in Fig. 3a needs to be implemented only once: filtering and decimation is a common operation performed for all the satellite signals available.

The decimation strategy discussed above has been implemented in Matlab and used to process the cosine BOC(15, 2.5) signal broadcast by the GIOVE-B satellite. When a down-sampling
factor, \( K = 4 \), is used a 50% speed increase is obtained with respect to standard processing. This result is very conservative since it was obtained considering a single signal. In this way, the computational cost of down-sampling and decimation was not shared among several signals. More details on the results obtained are provided in Section 6.

4 Theoretical characterisation

In this section, CSB processing is characterised in terms of coherent output SNR and tracking jitter. The former measures the signal quality at the correlator outputs whereas the latter is the normalised standard deviation of the residual error at the output of a tracking loop. These two metrics are analysed in the following sections.

4.1 Coherent output SNR

The coherent output SNR is computed at the output of the prompt correlator under the assumption of perfect signal recovery, i.e. when the signal parameters estimated by the tracking loops match those of the incoming signal. In particular, when a tracking loop adopting a single prompt correlator is used, the coherent output SNR is defined as [10, 16, 21]

\[
\text{SNR}_q = \max_{\varphi \in [0, 2\pi]} \frac{|E(P)|^2}{(1/2)\text{Var}(P)}.
\]

(16)

The factor (1/2) in (16) accounts for the fact that \( P \) is a complex quantity and only the power of its real part is considered. As discussed in Section 3.2, side-band prompt correlators, (13), can be coherently combined after carrier and subcarrier phase recovery. In this way, single equivalent correlator (15) is obtained. The coherent output SNR should thus be evaluated with respect to (15) as

\[
\text{SNR}_q = \max_{\varphi \in [0, 2\pi]} \frac{|E(P' + P')|^2}{(1/2)\text{Var}(P' + P')}.
\]

(17)

Using this definition and models (10) and (13), it is possible to evaluate the coherent output SNR for the CSB which is given by

\[
\text{SNR}_q = \frac{C \varphi^2}{(1/2)2\sigma_d^2} = \frac{C}{(1/N)\sigma_d^2} \frac{\varphi^2}{2} = \frac{C}{(1/N)N\sigma_d^2} \frac{\varphi^2}{2}.
\]

(18)

This is the same coherent output SNR achieved by the DPE [10]. As for the DPE, the term \( \varphi^2 / \sigma_d^2 \) is the square modulus of the correlation between the input filtered subcarrier and an equivalent subcarrier obtained by summing \( s_n[n] \) and \( s_n[n] \). As for the DPE, this term can lead to a loss with respect to the DE which, in the worst case, is equal to

\[
L_o = \frac{s^2}{2} = \frac{8}{2} \Rightarrow L_o|_{\text{dB}} = -0.91 \text{ dB}.
\]

(19)

This SNR loss can become a gain in the presence of front-end filtering. The interested reader can find the SNR loss evaluated as function of the front-end bandwidth in [10]: since the CSB achieves the same performance of the DPE in terms of coherent output SNR, the results obtained for the DPE in [10] are also valid for the CSB. Thus, the coherent output SNR is not further analysed here.

The equivalence between CSB and DPE, in terms of coherent output SNR, will be empirically verified in Section 4 when real-data processing is discussed.

4.2 Tracking jitter

The tracking jitter measures the residual amount of noise present at the output of a tracking loop [22]. In particular, it can be evaluated as [22]

\[
\sigma_j = \frac{\sigma_d}{G_d} \sqrt{\text{SNR}_q}.
\]

(20)

where \( \sigma_d \) is the standard deviation of the discriminator output, \( B_{eq} \) is the loop equivalent bandwidth and \( T_c \) is the coherent integration time defined in (9). \( G_d \) is the discriminator gain defined as

\[
G_d = \frac{\partial E[S(\varphi)]}{\partial \varphi}_{\varphi = 0}.
\]

(21)

where \( S(\cdot) \) is the discriminator input–output function. \( S(\cdot) \) depends on the type of discriminator adopted by the loop.

In the CSB approach, a double-phase discriminator is used. In particular, carrier and subcarrier tracking loops are coupled. The equivalent model of the CSB tracking loops is shown in Fig. 4 where the loops are represented with respect to the carrier and subcarrier phase. The upper and lower branches of the CSB loops, which correspond to the upper and lower correlator branches in Fig. 2, contain the information relative to the sum and difference of the carrier and subcarrier phases, respectively.

The model shown in Fig. 4 has been obtained by representing the processing described in Fig. 2 with respect to the parameters of interest and by adopting linear approximations for the non-linear elements of the loops, i.e. the loop discriminators. This model is thus an approximation which is used only to derive the tracking
jitter. Since all the elements of the model are linear, linearity can be used to separate carrier and subcarrier processing paths. In particular, the blocks in dark grey before the definition of upper and lower branches and before the "NCO model" elements perform a linear transformation which is the inverse of the operation performed by the block inside the light grey box placed before the injection of the equivalent carrier and subcarrier noise terms. Thus, the model can be significantly simplified under the assumption that the same linear 'NCO model' is used for both carrier and subcarrier components. The blocks in grey in Fig. 4 can be removed and the analysis is conducted only for the subcarrier component:

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Using (22), it is possible to express the variance of $\Delta \phi_{sb}$ as

$$
\sigma_{\Delta \phi_{sb}}^2 = \frac{1}{2} \text{Var}\{\Delta P^*\} = \frac{1}{2} \text{Var}\{\angle P^* - \angle P\}
$$

where the equality holds true in the absence of phase wrapping effects, i.e. when $|\angle P^* - \angle P| < \pi$.

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$$
\sigma_{\Delta \phi_{sb}}^2 = \frac{1}{2} \text{Var}\{\angle P^* - \angle P\}
$$

This result follows from the fact that the noise terms of $P^*$ and $P$ are i.i.d. When the signal parameters are correctly tracked, the SNR associated to the upper and lower correlators is equal to half the coherent output SNR evaluated in Section 4.1. Using this result, the tracking jitter can be computed using the approach developed in [24] for the evaluation of the normalised standard deviation at the output of an arctangent discriminator. In particular, it is possible to show that

$$
\sigma_{\Delta \phi_{sb}} = \frac{1}{\sqrt{2}} \sqrt{\frac{1}{4} \text{SNR}_{sb} + \frac{1}{4} \text{SNR}_{eq}} = \frac{1}{\sqrt{2}} \sqrt{\frac{1}{4} \text{SNR}_{sb} + \frac{1}{4} \text{SNR}_{eq}}
$$

Replacing $\text{SNR}_{sb}$ with its expression in (18), the normalised discriminator standard deviation becomes

$$
\sigma_{\Delta \phi_{sb}} = \sqrt{\frac{1}{\text{SNR}_{sb}} \left[ 1 + \frac{2}{\text{SNR}_{eq}} \right]} = \sqrt{\frac{1}{\text{SNR}_{sb}} \left[ 1 + \frac{1}{\text{SNR}_{eq}} \right]}
$$

and the tracking jitter assumes the following form:

$$
\sigma_j = \frac{1}{2} \sqrt{\frac{2B_{eq} \pi}{N/C(N/B_{Rx})}} \left[ 1 + \frac{1}{N(C/N/B_{Rx})} \right] \text{[rad]}
$$

Finally, assuming that the receiver bandwidth, $B_{Rx}$, is approximately equal to half the sampling frequency

$$
B_{Rx} \approx \frac{f_s}{2}
$$

the tracking jitter becomes

$$
\sigma_j = \sqrt{\frac{B_{eq} \pi C}{N/C(N/B_{Rx})}} \left[ 1 + \frac{1}{N(C/N/B_{Rx})} \right] \text{[rad]}
$$

The tracking jitter in (28) is expressed in units of radians and it can be easily expressed in metres through the multiplication by

$$
\frac{c \cdot f_{sub}}{2 \pi}
$$

where $c$ denotes the speed of light. The validity of (28) is investigated in Section 5 where simulations are used to support theoretical findings. Results are presented in metres since tracking jitter is usually provided using this unit of measurements. An expression similar to (28) can be found for the carrier component.

The tracking jitter for the DPE was determined in [10] and it is given by

$$
\sigma_{j_{DPE}} = \frac{1}{2} \sqrt{\frac{B_{eq} \pi C}{N/C(N/B_{Rx})}} \left[ 1 + \frac{1}{2(N/C(N/B_{Rx})} \right] \text{[rad]}
$$

Expression (30) differs from (28) for the presence of an additional factor '2' in the denominator of the last term under square root. This term is usually referred to as squaring loss and it is due to the non-linear nature of a tracking loop: when developing tracking loop linear approximations, an equivalent noise term is introduced. The variance of this noise term accounts for the non-linear effects of the loop discriminator and leads to the squaring loss. CSB processing using ML discriminators has a slightly higher squaring
loss with respect to the DPE. This effect however becomes negligible for low CN0 values and for long integration times.

4.3 Multipath error envelope (MEE)

The impact of multipath on a GNSS tracking algorithm is usually assessed using the MEE that is the area spanned by the estimation error induced on the subcarrier. For each case considered in the figure, three curves are present [27]:

- from the actual error: the tracking jitter obtained from the actual error can be observed.
- from the loop filter output: it has been obtained by estimating the standard deviation of the final subcarrier phase error.
- from the filter output: it has been obtained by propagating the standard deviation of the subcarrier loop filter output.

The last curve which was obtained using the loop filter output is important to support the validity of the linear model discussed in Section 4: since the standard deviation of the loop filter output was propagated using the model derived in Section 4, significant deviations from the theoretical curve may have indicated inconsistencies in the linear model developed.

A good agreement between the three curves is obtained for the three cases considered in Fig. 7 for CN0 values >26 dB Hz. This phenomenon is expected since, for low CN0 values, the loop becomes unstable and non-linear effects start to impact the behaviour of the system. For low CN0 values, the CSB loops lose lock and the tracking jitter diverges to infinity. This phenomenon can be clearly seen for the case $B_{0} = 5$ Hz where sudden jumps in the tracking jitter obtained from the actual error can be observed. Theoretical formula (28) is able to effectively predict the behaviour of the loop under lock conditions.
To verify the ability of the CSB approach to track BOC-modulated satellite. The dataset used for the analysis is the same employed in further supporting the theoretical findings described in Section 4.2. Configurations with different integration times, loop equivalent values >26 dB Hz the two approaches achieve similar performance.

In particular, a National Instruments (NI) PXI-e 5663 vector analyser was used to collect data from the GIOVE-B signals, real data collected using a radio frequency signal analyser. This is an open access article published by the IET under the Creative Commons Attribution -NonCommercial License.

Table 2: Parameters adopted for the collection of wide-band GIOVE-B signals

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>sampling frequency</td>
<td>f_s = 40 MHz</td>
</tr>
<tr>
<td>sampling type</td>
<td>complex I&amp;Q</td>
</tr>
<tr>
<td>centre frequency</td>
<td>1575.42 MHz</td>
</tr>
<tr>
<td>number of bits</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 3: Parameters adopted for the processing of the GIOVE-B signals

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSB carrier branch order</td>
<td>3</td>
</tr>
<tr>
<td>CSB carrier branch bandwidth</td>
<td>10 Hz</td>
</tr>
<tr>
<td>CSB subcarrier branch order</td>
<td>2</td>
</tr>
<tr>
<td>CSB subcarrier branch bandwidth</td>
<td>5 Hz</td>
</tr>
<tr>
<td>DLL order</td>
<td>2</td>
</tr>
<tr>
<td>DLL bandwidth</td>
<td>5 Hz</td>
</tr>
<tr>
<td>DLL early-minus-late chip spacing</td>
<td>0.5 chips</td>
</tr>
<tr>
<td>E1a integration time (before bit synch)</td>
<td>2 ms</td>
</tr>
<tr>
<td>E1a integration time (after bit synch)</td>
<td>10 ms</td>
</tr>
<tr>
<td>E1c integration time</td>
<td>8 ms</td>
</tr>
</tbody>
</table>

Fig. 7 also provides the tracking jitter achieved by the subcarrier phase lock loop (SPLL) of the DPE [10]: for C/N0 values >26 dB Hz the two approaches achieve similar performance further supporting the theoretical findings described in Section 4.2.

Similar results were obtained considering different configurations with different integration times, loop equivalent bandwidths and loop orders. A good agreement was found between theoretical and simulation results supporting the validity of the findings discussed in Section 4.

6 Real data analysis

To verify the ability of the CSB approach to track BOC-modulated signals, real data collected using a radio frequency signal analyser were used. In particular, a National Instruments (NI) PXI-e 5663 vector analyser was used to collect data from the GIOVE-B satellite. The dataset used for the analysis is the same employed in [10] to test the DPE. As mentioned in [10], the GIOVE-B satellite was decommissioned in the Summer 2012 and the dataset used in this paper was collected on 5 November 2011. The use of such dataset is justified by the fact that it contains valid cosine BOC(15, 2.5) data with a known pseudo-random noise code [28]. Thus, the use of this dataset allows one to test the CSB approach for both the sine BOC(1, 1) and cosine BOC(15, 2.5) modulations broadcast in the Galileo E1 band. The cosine BOC(15, 2.5) transmitted by the currently operating Galileo satellites is encrypted and requires the use of codeless techniques [29]. Moreover, the adoption of the same dataset used in [10] allows a direct comparison of the CSB with the DPE.

The parameters adopted for the data collection are reported in Table 2: the NI signal analyser has a bandwidth of about 38 MHz and is able to capture the main lobes of the cosine BOC(15, 2.5) modulation.

The GIOVE-B cosine BOC(15, 2.5) is characterised by a primary code with a 2 ms duration and by a secondary code of five elements. Thus, after secondary code recovery, the integration time can be extended from 2 to 10 ms. The processing implemented here integrates the received signal for 2 ms until secondary code synchronisation. The integration time is then extended to 10 ms. For the sine BOC(1, 1) signal, the integration time is kept constant to 8 ms, the duration of one code period.

The C/N0 is computed using the wide band versus narrow band (WBNB) C/N0 estimator [22] and C/N0 estimates are provided only after secondary code synchronisation. The estimation of the C/N0 is based on composite prompt correlator (15).

The parameters adopted for the processing of the GIOVE-B signals are provided in Table 3 and are the same parameters used in [10] for the DPE.

Results related to the processing of the GIOVE-B cosine BOC(15, 2.5) are shown in Fig. 8. Six correlators, the upper and lower early, late and prompt correlators, are evaluated and combined according to the architecture described in Section 3.

Figs. 8a and b show the magnitude of the upper and lower early, late and prompt correlators. The correlators of the two branches have a similar behaviour and, after an initial transient, the amplitude of the two prompt correlators is maximised. The same amplitude is observed in the upper and lower branches of the CSB algorithm. The real and imaginary parts of the upper and lower prompt correlators are shown in Figs. 8c and d. After a few epochs, the phases of the prompt correlators are properly recovered and most of the signal power is concentrated in the real part of the two correlators. After ~2 s, the secondary code is recovered and the
integration time is increased to 10 ms. The secondary code is removed and the navigation bits can be clearly identified in the real parts of the upper and lower prompt correlators shown in Figs. 8a and 8d. The real parts of the upper and lower prompt correlators have the same sign and thus can be coherently combined.

The \( C/N_0 \) estimated using the WBNB estimator is shown in Fig. 8e and the normalised code, carrier and subcarrier Doppler estimates are provided in Fig. 8f. Doppler estimates correspond to the outputs of the code, carrier and subcarrier loop filters. Each estimate is normalised by its fundamental frequency: the carrier Doppler is normalised by the GPS L1 centre frequency, 1575.42 MHz, the DLL filter output by the nominal code rate, 2.5575 MHz and the subcarrier Doppler by the subcarrier rate, 15.345 MHz. When normalised, the three components assume similar values indicating the proper operations of the CSB architecture. As for the DPE, this property can be used to perform carrier aiding, i.e. the normalised carrier Doppler can be used to aid the processing of the code and subcarrier components.

The \( C/N_0 \) estimates provided in Fig. 8e are further analysed in Fig. 9 which considers the second part of the GIOVE-B dataset and compares the \( C/N_0 \) estimated for the CSB processing scheme with that obtained for the DPE and DE. Details relative to the processing implemented for the DPE and DE can be found in [10].

From Fig. 9, it emerges that CSB processing achieves the same \( C/N_0 \) performance of the DPE and outperforms the DE for the specific conditions considered here. The GIOVE-B signal considered here is significantly filtered and CSB processing and DPE are able to provide improved performance in the presence of front-end filtering. The equivalence in terms of \( C/N_0 \) between CSB processing and DPE was expected and it is in agreement with the findings obtained in [10].

As for the previous case, the CSB approach achieves the same performance of the DPE. In this case, the DE outperforms the other two approaches and a loss of 0.61 dB is experienced. This result is expected and it is in agreement with the findings obtained in [10].

The sample results provided in this section illustrate the ability of the CSB to process different BOC signals and support the theoretical findings discussed in Section 4.

7 Conclusions

In this paper, a novel SBP approach is proposed. The algorithm, denoted as CSB, recombines the upper and lower side-band correlators computed in SBP to estimate the subcarrier parameters and to fully exploit the subcarrier benefits. The CSB approach establishes a connection between SBP and independent subcarrier tracking performed by the DPE and by the DE. In particular, CSB processing is equivalent to the DPE: CSB processing achieves unambiguous BOC tracking and it is able to provide subcarrier phase measurements. The availability of side-band correlators allows the implementations of different processing schemes, including the usage of different carrier and subcarrier discriminators. ML carrier and subcarrier discriminators were derived and used to extract carrier and subcarrier information. Finally, in CSB processing, the usage of a dedicated loop which
performs subcarrier correlation is no longer required. The CSB approach provides an alternative to this patented solution. The CSB was characterised from a theoretical point of view and theoretical findings were supported by simulations and real-data processing. Real wide-band GNSS signals from the GIOVE-B satellite were used to demonstrate CSB processing. The analysis confirms that the CSB approach is an effective alternative to independent subcarrier tracking.

8 References


![Fig. 9](image1.png) *C/N₀ estimates obtained using CSB approach (with and without signal decimation), DPE and DE for the GIOVE-B cosine BOC(15, 2.5) signal. CSB processing achieves the same C/N₀ of the DPE and outperforms the DE with a C/N₀ improvement of about 0.87 dB.*

![Fig. 10](image2.png) *C/N₀ estimates obtained using the CSB approach, the DPE and DE for the GIOVE-B sine BOC(1, 1) signal. CSB processing and DPE have the same performance and experience a 0.61 dB loss with respect to the DE.*
In this appendix, the ML discriminators for binary offset carrier (BOC) modulation are derived. In particular, the only terms in (33) which depend on $\Delta \varphi_b$ and $\Delta \varphi$ are

$$L_c(\Delta \varphi, \Delta \varphi_b) = \Re \left\{ \left[ p^* e^{j(\Delta \varphi + \Delta \varphi_b)} + \Re\{ p e^{j(\Delta \varphi - \Delta \varphi_b)} \} \right] \right\}$$

(35)

where

$$\varphi^* = \angle p^*, \quad \varphi = \angle p$$

(36)

Scale log-likelihood function (35) is maximised when its derivatives with respect to $\Delta \varphi$ and $\Delta \varphi_b$ are equal to zero.

$$\frac{\partial L_c}{\partial \Delta \varphi} = p^* \sin(\varphi^* - \Delta \varphi - \Delta \varphi_b) + p \sin(\varphi - \Delta \varphi + \Delta \varphi_b) = 0$$

(37)

$$\frac{\partial L_c}{\partial \Delta \varphi_b} = -p^* \sin(\varphi^* - \Delta \varphi - \Delta \varphi_b) - p \sin(\varphi - \Delta \varphi + \Delta \varphi_b) = 0$$

(38)

Conditions (37) are met for

$$\Delta \varphi = \frac{1}{2} \angle \varphi^* p^*$$

and

$$\Delta \varphi_b = \frac{1}{2} \angle \varphi p$$

(39)

which are the ML discriminator for BOC carrier and subcarrier phase discriminators. Equation (38) maximises cost function (35) which assumes its maximum value, $|p|^2 + |p|$. This value is obtained when both cosine terms in (35) are equal to 1.

As mentioned above, discriminators are independent from the navigation bin, $d$, which is removed through the multiplication between the two side-band prompt correlators, $p^*$ and $p$.

### 9.2 Appendix 2: Subcarrier multipath error envelope

In this appendix, the SMEE for the CSB technique is derived using an approach similar to that adopted in [11] for the DPE. In particular, it is possible to show that, in the presence of a multipath ray, also correlator outputs (13) are given by the combination of a LOS and a reflected signal component (see (39)) where $A = (\sqrt{Cs_0}/2)d$.

$$p^* = a_0 e^{j(\Delta \varphi + \Delta \varphi_b)} + 1 + \alpha e^{j\omega} e^{j2\pi f_0 t_a} + \eta_a$$

(40)

where

$$a_0 = d A R(t - r_0)$$

(41)
Note that the assumption on $\tilde{R}(\cdot)$ is generally made in the literature [19, 22, 25] to analyse separately the impact of multipath on the code and carrier components. The same approach is adopted here for the analysis and derivation of the SMEE.

The SMEE describes the error induced by a multipath component and is obtained by evaluating the subcarrier discriminator output in the absence of noise. In this case, the subcarrier discriminator output is given by (14) and involves the multiplication of the two prompt correlators which, in the absence of noise, is given by (see (42) and (43)) The subcarrier phase estimated by discriminator (14) is given by (see (43)) where the term in square brackets is the multipath error expressed in radians

$$\text{ME}(\phi_1, \tau_1) = \frac{1}{2} \angle \left[ 1 + \alpha e^{j \phi_1} e^{-j 2 \pi f_{sub} \tau_1} \right] + \angle \left[ 1 + \alpha e^{-j \phi_1} e^{j 2 \pi f_{sub} \tau_1} \right].$$

(44)

The SMEE is the region spanned by the multipath error when varying the multipath excess phase, $\phi_1$, and it is contained within the maximum and minimum multipath error. The maxima and minima of (44) with respect to $\phi_1$ are obtained for

$$\phi_1 = 0, \pi.$$  

(45)

This condition leads to

$$\text{ME}_{\phi_1}(\tau_1) = \angle \left[ 1 \pm \alpha e^{-j 2 \pi f_{sub} \tau_1} \right] = - \arctan \left( \frac{\pm \alpha \sin(2 \pi f_{sub} \tau_1)}{1 \pm \alpha \cos(2 \pi f_{sub} \tau_1)} \right)$$

(46)

from which (32) directly follows. The SMEE is the area within the two curves defined in (46) and it is the same obtained in [11] for the DPE. This shows that the two approaches have the same performance in terms of SMEE.