Comment on ‘Strong recovery conditions for least support orthogonal matching pursuit in noisy case’

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In a very interesting recent paper, the authors proposed a least support denoising-orthogonal matching pursuit (LSD-OMP) algorithm and gave a sufficient condition for LSD-OMP algorithm in terms of restricted isometry constant of the measurement matrix. According to Dai and Milenkovic (2009), Wang et al. (2012) and Needell and Tropp (2009), this comment will point out that there are some errors in Tawfic and Kayhan (2015).

Introduction: A least support denoising-orthogonal matching pursuit (LSD-OMP) was proposed in [1–4]. Given y and Φ, it aims to recover a K-spars signal x from the following noisy measurement:

\[ y = Φx + e \]

where \( Φ \in \mathbb{R}^{m \times n} \).

To clarify our reasons, we list the LSD-OMP algorithm that is proposed in [1] as follows. We make no changes for this algorithm.

Algorithm 1 Least support algorithm

Input: measurement matrix \( Φ_{m \times n} \), measurement signal \( y_{m \times 1} \), sparsity \( K \), least support parameter \( L \), stop condition.

Initialisation: index set \( I_0 = \emptyset \), initial least support set \( J_0 = \emptyset \), iteration \( ℓ = 0 \), residual signal \( r_0 = y \).

Repeat steps 1–3 for \( L \) iterations or until stop condition

1: (update least support set)

\[ J_ℓ = \arg\max_{r_{ℓ-1} \in I_{ℓ-1}} |r_{ℓ-1} | \]

2: (reconstruction) \( ˆx = Φ_{J_ℓ}^\dagger y \)

3: (residual update) \( r_ℓ = y - Φ_{J_ℓ} ˆx \)

Output: recovered sparse signal \( ˆx \) with \( J_ℓ \), \( r_ℓ \) and \( ˆx_{J_ℓ} = 0 \)

Some errors in [1]: In this section, we will present some errors in [1].

- In the second paragraph of Section 1 ‘Introduction’ in [1]. The authors pointed out that ‘Unlike the standard OMP algorithms, the LSD-OMP does not require knowledge of sparsity \( K \).’ This sentence is not accurate. In the first paragraph of Section 2, ‘sufficient conditions for LSD-OMP’, the authors said ‘The initial value of \( L \) is chosen as min \((K/2, m/16)\) and when the stop...’. According to Algorithm 1, the inputs contain sparsity \( K \). So, the LSD-OMP requires knowledge of sparsity \( K \).


- In the first paragraph of Section 2, ‘Sufficient conditions for LSD-OMP’, the authors said ‘The LSD-OMP algorithm selects L atoms by finding the maximum correlation between \( Φ \) and residual vector \( r \), at each iteration and then adds them to the least support set. The set is used to estimate \( ˆx \) and to update \( r \). This process is repeated \( L \) times or until the stop condition of the LSD-OMP is reached’. Firstly, the authors did not show clearly the symbol \( ˆx \). Second, suppose that the symbol \( ˆx \) is the measurement matrix \( Φ \). Then the LSD-OMP is similar as generalised OMP (GOMP) presented in [4]; however, in Section 3 ‘Experimental results’, the authors did not compare LSD-OMP with GOMP.

- In the proof of Theorem 1 in [1], the authors said ‘Here, \( T_0 \) represents the support of the whole signal, \( T_2 \) represents a part of the support with size \( L \), which is used as a least support set to reconstruct the original signal ...’. Then we have \( T_2 \subseteq T_0 \) and \( |T_2| = L \). (2) is not used in the proof of Theorem 1. The inequality (3) in [1] is not accurate. (3) should be

\[ \|Φ_{T_0} y_0\| = \|Φ_{T_2}^\dagger Φ_{T_2} x_{T_2} + Φ_{T_2}^\dagger e\| \]

\[ \leq \|Φ_{T_2}^\dagger Φ_{T_2} x_{T_2}\| + \|Φ_{T_2}^\dagger e\| \]

\[ \leq (1 + δ_K).\|x_{T_2}\| + \sqrt{1 + δ_K.\|e\|} \]

(4) is not accurate and it should be

\[ \|Φ_{T_0} y_0\| = \|Φ_{T_2}^\dagger Φ_{T_2} x_{T_2} + Φ_{T_2}^\dagger e\| \]

\[ \geq \|Φ_{T_2}^\dagger Φ_{T_2} x_{T_2}\| - \|Φ_{T_2}^\dagger e\| \]

\[ \geq (1 - δ_K).\|x_{T_2}\| - \sqrt{1 + δ_K.\|e\|} \]

- Following the steps in the proof of Theorem 1, the inequality (5) in [1] is not accurate. In fact, the sentence ‘Substituting (2) into (1) yields’ should be ‘Substituting (4) and (3) yields’. (5) should be

\[ (1 - δ_K).\|x_{T_2}\| \leq (2\sqrt{1 + δ_K} + \sqrt{1 + δ_K.\|e\|}) \]

In fact, the inequality (7) in [1] is unreasonable. In noiseless case, \( \|e\| = 0 \), then according to (7), we will get \( |x_{T_2}| \leq 0 \). This is unreasonable. (7) is used for the proof of Theorem 1, so the proof of Theorem 1 is not accurate.

- The Lemma 1 in [1] cannot be used for the proof of Theorem, due to the result in Lemma 1 can be obtained under noiseless case. However, the authors consider the noisy case.

- The inequality (9) in [1] is not accurate and it should be

\[ \|y_{T_2}\| \leq \sqrt{1 + δ_K}.\|x_{T_2}\| + \|e\| \]

The authors did not give the mean of \( y_{T_2}, y_{T_3}, y_{T_4}, y_{T_5}, y_L \). There are many symbols should be explained.

- The sentence ‘Substituting (17) into (16) gives’ should be ‘Substituting (17) into (14)’.

- In the step 1 of Algorithm 1, the equality

\[ J_ℓ = \arg\max_{r_{ℓ-1} \in I_{ℓ-1}} |r_{ℓ-1} | \]

may be inaccurate.

Conclusion: In this paper, some errors in [1] is presented.

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