Crosstalk Increase in Tightly Bent Multi-Core Fiber Due to Power Coupling Mediated by Cladding Modes

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Abstract We developed simplified coupled-power equations including cladding modes, and revealed that crosstalk in tightly bent multi-core fibers can be increased from the existing predictions. The crosstalk increase was proportional to the square of the bend loss coefficient, and validated by experimental results, for the first time.

Introduction Spatial division multiplexing (SDM) using a multi-core fiber (MCF) is a strong candidate technology to overcome the capacity limit of single-core fiber transmission systems1. Inter-core crosstalk (XT) is one of the most important properties for SDM transmission systems that utilize each core of the MCF as an individual spatial channel. Characteristics of the XT have been actively studied, and significant dependence of the XT on the fiber bend and structure has been elucidated theoretically and experimentally2–5. However, the measurement results have been reported only for the loose bend, and there are no results for the tight bend. In this paper, we report on the unique XT behavior in the tightly bent MCF. We speculated that the cladding modes may involve the power coupling in a tightly bent MCF, and developed novel simplified coupled-power equations including the cladding modes. From the equations, we found that the XT can be increased by the tight bend in proportion to the square of the bend loss coefficient, which was also validated by the experimental results.

Crosstalk between cores in the MCF Since MCFs are perturbed by longitudinal variations and fluctuations of bend, twist, and fiber structure, the XT is stochastic parameter. However the statistical average/mean of the XT (mean XT) in the MCF is accumulated by obeying the coupled-power equation (CPE)6:

$$\frac{dP_n}{dz} = \sum_{m \neq n} h_{nm} P_m - \sum_{m \neq n} h_{mn} P_n$$

$$= \sum_{m \neq n} (P_m - P_n) h_{mn}, \quad (\because \ h_{mn} = h_{nm}), \quad (1)$$

where $P_n$ is the power in Core $n$, and $h_{nm}$ is the power-coupling coefficient (PCC) from Core $m$ to Core $n$. The PCC for the MCF has been elucidated significantly by various groups2–5.

However, since Eq. (1) is the PCE only for direct couplings between core modes, and since the bend loss can be understood as the power coupling from core modes to the cladding modes in a bent fiber, Eq. (1) is considered to be inapplicable to a tightly bent MCF if the bend losses of its cores are large. Therefore, in this paper, we investigate the CPE that includes the powers of the cladding modes and reveal the XT behavior in the tightly bent MCF.

Coupled-power equations including cladding modes The CPE with the cladding modes can be expressed as

$$\frac{dP_{Core n}}{dz} = \sum_{m \neq n} h_{Core n,Core m} P_{Core m} - \sum_{m \neq n} h_{Core m,Core n} P_{Core n} + \sum_{l} h_{Core n,Clad l} P_{Clad l} - \sum_{l} h_{Clad l,Core n} P_{Core n}$$

$$= \sum_{m \neq n} (P_{Core m} - P_{Core n}) h_{Core m,Core n} + \sum_{l} (P_{Clad l} - P_{Core n}) h_{Clad l,Core n}, \quad (2a)$$

$$\frac{dP_{Clad l}}{dz} = \sum_{m} h_{Clad l,Core m} P_{Core m} - \sum_{m} h_{Core m,Clad l} P_{Clad l} + \sum_{k \neq l} h_{Clad l,Clad k} P_{Clad k} - \sum_{k \neq l} h_{Clad k,Clad l} P_{Clad k}$$

$$- \alpha_{Clad l} P_{Clad l}, \quad (2b)$$

where $P$ is the power guided by each mode, and the subscripts of Core $n$ and Clad $l$ represents Core mode $n$ and Cladding mode $l$, respectively, and $\alpha_{Clad l}$ is the additional power loss coefficient of Cladding mode $l$ from the core modes, which can include the losses due to scattering and absorption in the coating, and due to the radiation outside of the fiber. The attenuation coefficients of the core modes were assumed to be equal and thus omitted in Eqs. (2a) and (2b). In this paper, we deal with the powers and PCCs that are statistically averaged over twist angle of the MCF, and call “the modes that are not guided by the cores” the cladding modes, which include the modes propagating in the coating.

If we neglect the recoupling of the power from the cladding to the core, the bend loss coefficient [1/unit length] of Core $n$ is understood as the sum of the power coupling coefficients [1/unit length] from Core $n$ to all cladding modes.
in the bent condition, and represented as
\[
\alpha_{\text{bend}} = \sum \alpha_{\text{Clad}, \text{Core } n}. \tag{3}
\]

The bend loss coefficient in [dB/unit length] can be obtained by simply multiplying 10/ln10 to Eq. (3).

However, it is very difficult and impractical to derive all of the PCCs related to cladding modes and loss coefficients of all cladding modes; therefore, we assume the countless cladding modes as one continuous imaginary mode. Then, Eqs. (2a) and (2b) can be simplified as
\[
\frac{dP_{\text{Core } n}}{dz} = \sum m \alpha_{\text{Core } n, \text{Core } m, \text{Core } n} P_{\text{Core } m} - \sum h_{\text{Core } n, \text{Core } m, \text{Core } n} P_{\text{Core } m} + \sum m \alpha_{\text{Clad}(m)} P_{\text{Clad}(m)} - \sum h_{\text{Clad}(m)} P_{\text{Core } n} \tag{4a}
\]
and
\[
\frac{dP_{\text{Clad}(m)}}{dz} = h_{\text{Clad}, \text{Core } m} P_{\text{Core } m} - \left( \sum i \alpha_{\text{Clad}(i)} + \alpha_{\text{Clad}(m)} \right) P_{\text{Clad}(m)} \tag{4b}
\]
where \(P_{\text{Clad}(m)}\) is the power in the cladding modes coupled from Core mode \(m\), \(h_{\text{Core } i, \text{Clad}(m)}\) the PCC from \(P_{\text{Clad}(m)}\) to \(P_{\text{Core } i}\) and \(\alpha_{\text{Clad}(m)}\) the loss coefficient of \(P_{\text{Clad}(m)}\), respectively. Since the power distribution in the cladding may depend on the core position from which the power coupled to the cladding, we treat the powers coupled from different cores individually, thus introducing the notations of \(P_{\text{Clad}(m)}\), \(h_{\text{Core } i, \text{Clad}(m)}\), and \(\alpha_{\text{Clad}(m)}\).

If we assume that the bend is longitudinally constant and that \(\alpha_{\text{Clad}} >> h\), the power coupling to the cladding, and power loss in the cladding can be nearly counterbalanced—not completely because the system is not loss-less--; therefore \(dP_{\text{Clad}(m)/dz} = 0\) can hold. Accordingly, \(P_{\text{Clad}(m)}\) can be obtained as:
\[
P_{\text{Clad}(m)} \approx \frac{h_{\text{Clad}, \text{Core } m} P_{\text{Core } m}}{\sum i \alpha_{\text{Clad}(i)} + \alpha_{\text{Clad}(m)}}. \tag{5}
\]

By substituting Eq. (5) to Eq (4a), we obtain
\[
\frac{dP_{\text{Core } n}}{dz} \approx \sum m \alpha_{\text{Core } n, \text{Core } m, \text{Core } n} P_{\text{Core } m} - \sum h_{\text{Core } n, \text{Core } m, \text{Core } n} P_{\text{Core } m} + \sum m \alpha_{\text{bend } m} P_{\text{Core } m} - \sum h_{\text{bend } m} P_{\text{Core } n} \tag{6}
\]
where
\[
\alpha_{\text{bend } n} = h_{\text{Clad}, \text{Core } n}. \tag{7}
\]

In normal CPEs, the reciprocity of the PCC—\(h_{\text{Clad}, \text{Core } n} = h_{\text{Core } n, \text{Clad}}\)—holds for the conservation of power; however, in Eqs. (4a) and (4b), the reciprocity of the PCC does not hold, because the cladding modes are treated as the one imaginary mode—the PCC from the imaginary cladding mode to a core depends on the positions of cores, from which the power is coupled to the cladding, and to which the power is coupled from the cladding, since the power distribution of the imaginary mode depends on the core position from which the power is coupled to the cladding, as we mentioned above. Nevertheless, we may assume
\[
h_{\text{Core } n, \text{Clad}(m)} = h_{\text{Core } n, \text{Clad}(m)} \gamma_{nm} = \alpha_{\text{bend } n} \gamma_{nm} \tag{8}
\]
where \(\gamma_{nm}\) represents the correction factor for the PCC from the imaginary cladding mode to Core \(n\), for the power originated in Core \(m\). \(\gamma_{nm}\) can correct the PCC dependence on the core positions due to the bias of the power distribution in the cladding. Thus, Eq. (6) can rewritten as
\[
\frac{dP_{\text{Core } n}}{dz} \approx \sum m \alpha_{\text{Core } n, \text{Core } m, \text{Core } n} P_{\text{Core } m} - \sum h_{\text{Core } n, \text{Core } m, \text{Core } n} P_{\text{Core } m} + \sum \alpha_{\text{bend } n} \gamma_{nm} \alpha_{\text{bend } m} P_{\text{Core } m} - \sum \alpha_{\text{bend } n} P_{\text{Core } n} \tag{9a}
\]
where
\[
\gamma'_{nm} = \gamma_{nm} / \left( \sum \alpha_{\text{bend } i} \gamma_{im} + \alpha_{\text{Clad}(m)} \right). \tag{9b}
\]

\(\gamma'\) can be effectively independent of \(\alpha_{\text{bend}}\) if \(\alpha_{\text{bend}} << \alpha_{\text{Clad}}\) holds, and \(\alpha_{\text{bend}} << \alpha_{\text{Clad}}\) may hold in general fibers in which the light propagations in the cladding modes are suppressed.

From here, we discuss the case where two identical cores are coupled with low XT, for further simple description. \(P_{\text{Core } m} = P_{\text{Core } n}\) can be approximated as \(P_{\text{Core } m}\), since the XT is low. Since the bend losses are equivalent between the identical cores, the last term \((-\alpha_{\text{bend } m} P_{\text{Core } n})\) in the right-hand side of Eq. (9a) can be omitted for XT consideration. Thus, Eq. (9a) can be reduced to
\[
\frac{dP_{\text{Core } n}}{dz} \approx \left( h_{nm} + \gamma'_{nm} \alpha_{\text{bend } m} \right) P_{m}. \tag{10}
\]

Let \(L\) the total length of the MCF, and \(L_{\text{bend}}\) the length of the tightly-bent part of the MCF, the mean XT from Core \(m\) to Core \(n\) can be expressed as
\[
\overline{X}_{nm} = P_{m} / P_{m} \approx h_{nm} L + \gamma'_{nm} \alpha_{\text{bend } m} L_{\text{bend}}. \tag{11}
\]
Therefore, the XT increase \(\overline{X}_{IC, nm}\) from the indirect coupling due to the tight bend can be derived as
\[
\overline{X}_{IC, nm} = \gamma'_{nm} \alpha_{\text{bend } m} L_{\text{bend}}. \tag{12}
\]
in the linear scale, and as

$$X_{\text{IC}}^{\text{db}} = 10\log_{10}\left(1 + \frac{y'\alpha_{\text{bend}}^2}{h_{\text{nm}}L}\right),$$  \hspace{1cm} (13)

in the decibel scale.

**Measurement results**

To validate the discussion in the previous section, we evaluated the XT increase from the tight bend with an actual MCF. We fabricated an MCF with identical step-index cores. The core Δ was ~0.35%, the core diameter was ~8.9 μm, and the core pitch was ~45 μm.

Figure 1 shows the measurement setup for measuring the XT increase due to the tight bend. The XT increase was evaluated with the XT increase from the XT measured in the reference condition [see Fig. 1(a)] to the XT measured in the tightly bent condition [See Fig. 1(b)]. The XT was measured as the mean XT, averaged over 10-nm wavelength span from the XT spectra obtained by using a narrow-linewidth tunable laser source and synchronized power meter3,7. The light was launched to and received from the MCF cores by using trench-assisted single-mode fibers (TA-SMFs). The total length of the MCF was 22 m. The length of the tight-bend-applied part was 2 m, and most of the rest was loosely bent ($R_b = 140$ mm) and arranged after the 2-m-long tightly-bent portion to suppress the power in the cladding modes—the power received at the midpoint between the cores in the end facet of the MCF was low enough even when the light was incident at the midpoint of the cores at the another end. The XT increases were measured at the tight bend radii $R_b$ of 7.5 mm and 10 mm, and at the wavelengths from 1540 nm to 1625 nm with the step of 5 nm, for measuring at various levels of $\alpha_{\text{bend}}$. The measured mean XT for the MCF without the tight bend was less than the measurement floor, which was in the order of $10^{-7}$ for this measurement; therefore the variation of the XT from direct coupling, proportional to the bend radius3–5, is negligible.

Figure 2 shows the measurement results for the XT increase due to the tight bend. The solid circles represent the measurement results, and the solid line represents the linear regression using Eq. (12) with the fitting parameter of $y'$. In this case, $y'$ was fitted to be $-4.0 \times 10^{-5}$ [m]. Since Fig. 2 is a double logarithmic plot, only the relationship of $y = ax^b$ can be the straight line, and the slope of the line is $\log_{10}a/b$; therefore, it was clearly confirmed that the XT increase due to the tight bend is proportional to the square of the bend loss coefficient $\alpha_{\text{bend}}$.

**Conclusion**

We developed simplified coupled-power equations including cladding modes, and revealed that XT in the tightly bent MCF can be increased from the existing predictions that are derived only from the direct power couplings between the core modes. Based on the newly derived equations, the XT increase due to the tight bend was found to be proportional to the square of the bend loss coefficient, which was also validated by the experimental results. Therefore, in applications where MCFs can be tightly bent, suppression of the bend loss may be important not only for the loss suppression itself but also for XT suppression.

**References**