Appendix B
Signal Strength in the Ether Waves

It is common practice (including throughout this book) to state the signal strength of transmitted signals in dBm, even though this makes no physical sense. dBm is a unit of electrical power, a ratio of the signal power to one milliwatt. Power is defined only within a circuit. After transmission from an antenna, signals are rigorously defined only in terms of field strength. The correct units are volts per meter (or more often microvolts per meter).

However, in many communication theory applications, it is extremely convenient to define a transmitted signal at some point in space in terms of dBm. That definition really assumes the situation shown in Figure B-1, in which an ideal unity gain antenna is located at the point in space being considered. The signal power in dBm is then the output of that ideal antenna at that location.

Quick Conversion Formulas

The following equation will allow you to convert field density in microvolts per meter directly to the equivalent signal strength in dBm.

\[ P = -77 + 20 \log(E) - 20 \log(F) \]

where,

- \( P \) = Equivalent signal power (dBm)
- \( E \) = Field strength (\( \mu \)V/m)
- \( F \) = Frequency (MHz)
To convert the signal strength back into the equivalent field density, use the formula:

\[ E = 10^{\left(\frac{P + 77 + 20 \log(F)}{20}\right)} \]

**Derivation**

In case you don’t believe these formulas, or have a particular need to derive something, here is the derivation of the first equation.

The signal strength (i.e., output power) from the ideal antenna in Figure B-1 is defined by the formula:

\[ P(\text{watts}) = \left(\frac{E(v/m)^2}{c^2}\right) A(\text{m}^2) \] \[ \frac{A(\text{m}^2)}{Z_0(\text{ohms})} \]

where,

- \( P \) = Equivalent signal power
- \( E \) = Field strength
- \( A \) = Effective antenna area
- \( Z_0 \) = Impedance of free space

The effective antenna area can be defined as a function of antenna gain by the following formula:

\[ A = \frac{G \lambda^2}{2\pi} = \frac{G c^2}{4\pi F^2} \]

and the two constants are:

\[ Z_0 \approx 120 \pi \text{ ohms} \quad c = 3 \times 10^8 \text{ m/sec} \]

Setting the antenna gain equal to unity (i.e., 0 dB) and plugging the antenna area expression into the power equation gives:

\[ P = \frac{(E^2)(c^2)}{(480 \pi^2)(F^2)} \text{ volts}^2 \text{ meter}^2 \text{ sec}^2(1/\text{sec})^2 \text{ ohms} \]

\[ = 1.8998 \times 10^{13} \frac{E^2}{F^2} \text{ watts} \quad \text{(Combining all Units)} \]

But this gives the signal strength in watts and requires that the field density be input in volts/meter and the frequency in Hertz (not the most commonly used units). Multiplying the constant by the three factors:

- \( 10^{-12} \text{ V}^2/\mu\text{V}^2 \)
- \( 10^{-12} \text{ MHz}^2/\text{Hz}^2 \) (The frequency term is on the bottom)
- \( 10^3 \text{ mW/W} \)
yields the expression:

$$P(\text{in mW}) = 1.8998 \times 10^{-8} \frac{E(\mu\text{V/m})^2}{F(\text{MHz})^2}$$

which, when converted to dB form using the formulas in Chapter 2 and rounding the constant to the nearest whole number becomes:

$$P = -77 + 20 \log(E) - 20 \log(F)$$

(Q.E.D., as some may say.)