Sensitivity and robustness in control systems theory and design

Mathematical models invariably give an imperfect description of physical systems, and, in any case, the parameters involved in such a description are often subject to variation and uncertianty. In the analysis stage it is therefore important to assess the sensitivity of the system stability and performance to parameter variations, and, in the design stage, it is desirable to minimise this sensitivity and thus aim for insensitive or robust control schemes.

The importance of such considerations has been recognised for some time now, and the appearance of a special issue dedicated to sensitivity and robustness is long overdue. The emergence of new modern control and multivariable control techniques has certainly rekindled interest in the subject, as demonstrated by recent publications [1, 2] and the number of workshops organised throughout the world. In response to this the IEE Professional Group C1 (Control Theory) organised a half-day Colloquium on 'Robustness in multivariable control system design' (October 1981) and a one-day Colloquium on 'Robustness and sensitivity in feedback systems' (September 1982), to be followed by the present special issue.

Like so many other aspects of control, the mathematical ideas behind sensitivity and robustness existed well before the subject took shape and form in the context of feedback analysis and design. Eigenvalue/eigenvector sensitivity to variations in the elements of a matrix, for example, is a subject studied as early as 1846 [3]. Numerical analysts took an interest in this topic in the early 1960s [4, 5] and it was not until the mid-1960s that this area of work was put in a control context [6], soon to be followed by a number of contributions to the various control journals.

Well before this, however, it was Bode [7] who laid the foundations to robust feedback design. Despite this major contribution, the subject went into virtual hibernation until the early 1960s, when Horowitz launched his frequency-response approach to robustness [8]. His pioneering work evolved into a quantitative feedback theory and led to a successful design method. It is therefore with pleasure that we include in this issue a contribution from Prof. Horowitz, who gives an account of the method as well as surveying its many recent developments. Through its engineering insight, numerous illustrative examples and comparative criticism of other modern control methods, the paper should stimulate debate. Further contributions in the same general area of work come from Dr. Ashworth and Prof. Towill, included in this special issue, and Dr. East [9].

The 1970s saw the extension of frequency-response methods to the case of multivariable systems. A significant development here is the generalisation of the Nyquist criterion [10] and the associated design method [11]. A first discussion of the use of singular values or principal gains in the study of sensitivity with respect to closed-loop accuracy was given [11], but it was not unitl a few years later that the subject received full attention with respect to stability and performance [1]. Since the late 1970s, the major preoccupation in this area has been with the derivation of less conservative results. The present special issue contains a number of contributions which provide effective answers. Thus by appropriate use of sector conditions Dr. Kouvaritakis and Dr. Postlethwaite propose an improvement on an earlier method [1]. Dr. Doyle defines a new function μ which exploits constraints on the structure of perturbations and deals with simultaneous input and output perturbations. Prof. Safonov also exploits structural properties of perturbations and introduces scaling in order to optimise the assessment of robust stability margins. Using related techniques, Dr. Limebeer and Dr. Hung [12] extend robust stability results to interconnected systems.

Singular-value techniques have also proved useful in the assessment of robustness properties of LQG regulators and in the selection of suitable weights in the index of performance [1]. Further developments in this area achieved by suitable use of the Bode gain/phase relationship are reported in this special issue in a paper by Dr. Daniel.

LQG design in the face of uncertainty is also the topic of Prof. Grimble's and Dr. Swierniak's papers. The latter uses state-space inequalities in order to take uncertainty into account and is thus able to assess the deterioration of performance. Prof. Grimble, on the other hand, considers models with constant but unknown parameters and constructs a Kalman filter on the basis of given mean values and covariances for the uncertain parameters.

Another area of design methods which has seen significant development in the last decade comes under the general heading of optimisation [1, 13]. Here, optimisation techniques have been proposed for the design of parameters of control schemes which retain the properties of stability, disturbance rejection and tracking in the presence of uncertainty. This problem, together with the additional constraint of decoupling, is solved by appropriate use of optimal interpolation results in Prof. Safanov and Dr. Chen's paper, included in this issue. In addition to this contribution, a presentation by Prof. Kwakernaak [14] reported on some recent developments which are based on a minimax optimisation of the sensitivity function.

The problem of robust control design for asymptotic tracking is approached from different points of view by Prof. Doraiswami and Prof. Porter. The former proposes a dual-mode control strategy in order to extend some of Davison's early work and thus avoids intersample ripple effects due to disturbances. Prof. Porter, on the other hand, deals with uncertainty in system delays and derives a steady-state design by an asymptotic analysis which allows the sampling period to become arbitrarily large. Uncertainty in delays is also dealt with in Dr. Owens and Mr. Raya's paper which examines the robust stability properties of Smith predictor feedback schemes.

Adaptive control techniques are well suited to deal with uncertainty and disturbances, and in recent years a good deal of effort has been devoted to the study of the robustness properties of adaptive schemes. This is precisely the topic of Dr. Cook and Mr. Chen's contribution, which considers model-reference schemes, and Dr. Gawthrop's presentation [15] which investigated the robustness of self-tuning regulators.

A number of fundamental results have been generalised to the multivariable case. This, as could be predicted, would not be the end, and indeed more recently 'classical and modern' control results have been extended to an even more complex class of systems: the large-scale systems. We are pleased to have a contribution in this area from Prof. Šiljak and co-workers, who propose a decentralised control scheme for the purpose of stabilising a large-scale system in the face of uncertainty in the parameters and nonlinearity in the interactions.

The problem of sensitivity and robustness, as is transparent from the collection of papers in this special issue, is multifaceted. Clearly, it would not be possible to include all aspects of the subject in a single issue. Inevitably some significant areas of research, such as the geometric approach to robustness and the problem of the robust servomechanism, are not represented in this issue. It is also inevitable that some other areas, such as adaptive control, large-scale systems and the LQG problem, despite their intrinsic interest, are underrepresented. The spectrum of ideas contained in this issue is nevertheless wide and highlights a number of important aspects.

As mentioned earlier, interest in this area of control is not new. By the early 1960s a substantial body of knowledge had begun to form and by 1964 it was possible to hold the first conference, the Symposium on Sensitivity analysis, in Dubrovnik, Yugoslavia. The extent of activity in the subject can be judged from the approximately 50 papers that were presented by authors from about 15 countries. The subject coverage can be seen from the sessions: namely, general problems of sensitivity analysis, sensitivity functions, sensitivity/stability, sensitivity/feedback, sensitivity/optimal systems and sensitivity/adaptive systems. Such was the enthusiasm and optimism of this era that, of the whole range of topics considered, only a few, such as large parameter variations and structural sensitivity of various functional block decompositions, were deemed not to have been resolved.

The recent resurgence of interest in the area has brought about a whole range of new ideas, but also new problems. Indeed, the present issue stands as proof of the number of exciting tasks already accomplished and also the important targets yet to be achieved. It is to be hoped, for example, that techniques for exploiting the structure of perturbations will be developed further; that the nonlinear nature of real-life systems will be taken more fully into account; that a variety of further design techniques aiming at prescribed performance tolerances will be developed; that such techniques will be appraised and compared with existing ones; that the problem of robustness in adaptive control will attract more research; and that new tools such as fuzzy logic may be introduced into robustness. We hope that the breadth and depth of the ideas presented in this special issue will stimulate discussion and thus make a contribution to the future development of sensitivity and robustness.

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B. KOUVARITAKIS
D.H. OWENS
PROF. M.J. GRIMBLE

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